Mathematics in a Second Grade Classroom:

The Effects of Cognitively Guided Problem Solving

by

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ABSTRACT

The need for improved mathematics education in many of America's schools that serve students from low income households has been extensively documented. This practical action research study, set in a suburban Title I school with a primarily Hispanic, non-native English speaking population, is designed to explore the effects of the progression through a set of problem solving solution strategies on the mathematics problem solving abilities of 2nd grade students. Students worked in class with partners to complete a Cognitively Guided Instruction-style (CGI) mathematics word problem using a dictated solution strategy five days a week for twelve weeks, three or four weeks for each of four solution strategies. The phases included acting out the problem using realia, representing the problem using standard mathematics manipulatives, modeling the problem using a schematic representation, and solving the problem using a number sentence. Data were collected using a five question problem solving pre- and post-assessment, video recorded observations, and Daily Answer Recording Slips or Mathematics Problem Solving Journals. Findings showed that this problem solving innovation was effective in increasing the problem solving abilities of all participants in this study, with an average increase of 63% in the number of pre-assessment to post-assessment questions answered correctly. Additionally, students increased the complexity of solutions used to solve problems and decreased the rate of guessing at answers to word problems. Further rounds of research looking into the direct effects of the MKO are suggested as next steps of research.

Key words: mathematics, problem solving, Cognitively Guided Instruction, realia, acting out, Vygotsky, constructivism, 2nd grade

DEDICATION

This dissertation is dedicated to my family, who, without their support, this process would have been insurmountable.

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CHAPTER 1

INTRODUCTION

Many believe education in the United States is in peril. The National Commission on Excellence in Education reported in 1983 that many minority and low socio-economic students were not learning at the same level as their more economically advantaged age-mates. This inequality in education has continued. American school children scored below the Organization for Economic Cooperation and Development average and ranked 35th out of 40 nations on the 2006 Program in International Student Assessment (PISA) of mathematics literacy, with America's Black and Hispanic students scoring well below our White and Asian students (Organization for Economic Cooperation and Development, 2008). The PISA assessment should not be considered an assessment of what each student has learned in just that school year, but, rather, it is an assessment of what the child has learned from birth (Berliner, 2011). The United States educational system's two main objectives are to develop each child's academic skills and to lower the achievement gap between races, genders, and socioeconomic groups (Konstantopoulos & Chung, 2011). Educationally disadvantaged children need to be given equal educational opportunities at school, including opportunities to learn how to solve problems through reasoning, evaluate decisions for soundness, persevere through difficulty, and communicate decisions with other people (Kilpatrick & Swafford, 2002; Lester Jr. & Charles, 2003; National Council of Teachers of Mathematics [NCTM], 2000, 2004; Sutton & Krueger, 2002). At home, parents are the proprietors of these experiences, guiding and providing a safe place for children to develop problem-solving skills. At school, teachers, through careful preparation, provide these growth-promoting experiences for children.

Mathematics class can offer an opportunity for children to explore problem solving in a non-threatening environment. When students are given opportunities to work through teacher selected problems in a community situation in which teachers facilitate, rather than take over the problem solving process, students are guided to understanding (Sutton & Krueger, 2002). This pedagogical approach relies on discovering and building mathematical relationships and helps students create mathematics understanding and knowledge (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Accepting that students can create their own understanding of mathematics, rather than it being taught to the student, allows connections to be made between what the student already knows and new mathematical concepts (National Research Council, 1989; Sutton & Krueger, 2002). A more knowledgeable other (MKO) in the classroom, such as a teacher or more advanced classmate who guides but does not tell, can serve as a scaffold, aiding the student to solve more complex problems than the student would have the cognitive ability to do independently (Vygotsky, 1978). When children are given the opportunity, through a teacher's careful allotment of time and problem selection, to construct new meanings from their past and current experiences, new understandings are formed (Brooks & Brooks, 1994; Dewey, 2002).

Problem solving is natural to young children because it is how they explore the world and develop new understandings. Teachers can use this natural inquisitiveness to foster students' learning in mathematics (NCTM, 2000). Problem solving typically develops through a set of incremental steps at the child's own developmental pace.

Initially, students solve word problems by representing each part of the problem using concrete objects, move on to using more sophisticated counting strategies that model the actions of the problem with concrete objects, and finally develop the use of arithmetic strategies, such as using known facts or derived facts (Carpenter & Moser, 1984; Fuson, 1988). Students deepen their understanding of mathematical concepts when they are allowed to create their own representations of problems, rather than following the teacher's mental representation of the problem (Schielack, Chancellor, & Childs, 2000).

Teachers can provide students with opportunities to reinforce their learning through verbalizing their reasoning and providing proof. Giving students opportunities to present their information to classmates, the teacher, or other active participants allows students to think through their solutions and justify their work (Carpenter et al., 1999; Kilpatrick & Swafford, 2002; Kline, 2008). Metacognition, making sense of one's understanding, has been shown to be successful in building transferable knowledge in students (Palincsar & Brown, 1984; Scardamalia, Bereiter, & Steinback, 1984; Schoenfeld, 1983, 1985, 1991, as stated in Bransford, Brown, & Cocking, 1999) and develops in classrooms where students are allowed to be inquisitive, share their thinking, and take risks (NCTM, 2000). Wu, An, King, Ramirez, and Evans (2009) found that by giving students the opportunity to share problems they have created and their solutions with the class, students' confidence in their mathematical abilities increases. In fact, further research has shown a significant positive relationship between attitude toward problem solving–confidence, patience, and willingness–and mathematics achievement (Mohd, Mahmood, & Ismail, 2011).

National Council of Teachers of Mathematics (NCTM) (2000) proposes that students should have daily opportunities to describe, discuss, and defend their thinking in mathematics. Discussing thinking gives students opportunities to develop appropriate mathematical vocabulary, deepen understanding of mathematical concepts, and think about alternate ways to solve problems (NCTM, 2000). In a recent study, Hartweg and Heisler (2007) used student discussion to allow the teacher to understand student thinking and use redirecting questions to clear up misconceptions, as well as to allow other students to question the problem solving strategies of their classmates. Their study found that even when students discovered another student's error, respect was shown, and the class worked as a whole to create mathematical understanding from the misconception. Partner and small group discussions create the opportunity for students to learn questioning techniques, justify work, and clear up misconceptions in a respectful, comprehension building manner (NCTM, 2000).

Evidence of the Problem

Working as a second grade general education teacher at San Marcos Elementary for the past six years, I have found that the majority of the second grade students in my classroom show weak problem solving skills, both in mathematical contexts and in their everyday lives. I see students confronted daily with problem situations, and oftentimes their choices of solutions are not in their best interest. For example, in his or her reaction to a lost paper or a parent's late arrival after school, a student could quickly become panicked, demonstrate illogical thought processes, or simply slip into physical or mental immobility. Likewise, when confronted with an unfamiliar problem in mathematics class some students react similarly, by shutting down mentally and immediately asking me for help or guessing at the answer. Because problem solving is an integral part of the Arizona Second Grade Mathematics Standards, weak problem solving skills impact nearly all areas of mathematics.

My second grade students show a general ability to solve basic word problems that require simple addition, but when problems require a procedure other than addition, have multiple steps, or contain irrelevant numbers or information, many students are unable to arrive at an accurate solution. On the Spring 2009 TerraNova assessment, only 48% of the second graders in my class scored at high mastery on the questions that assessed mathematics problem solving and reasoning. On the Spring 2010 Stanford Achievement Test Series Tenth Edition (Stanford 10), my second grade students averaged a national percentile scale score of 50% on mathematics problem solving, and on the Spring 2011 Stanford 10, my students averaged a national percentile scale score of 54%. Given that the Spring 2009 TerraNova Assessment and Spring 2010 and 2011 Stanford 10 assessment results and my own classroom observations indicate that a large percentage of the second graders I teach are unable to effectively solve mathematics problems using reasoning, increasing my students' mathematics problem solving skills was the focus of my action research.

Problem within the Local Context

Mathematics curriculum is dictated by the state but guided by the national standards. In the mid-2000s, 44 states agreed to work together to create a standardized document that set forth the learning goals in kindergarten through twelfth grade mathematics and English language arts education (Dacey & Polly, 2012). These standards, termed the Common Core State Standards, were designed to standardize educational goals throughout the country so graduates of the American school system would be prepared for college, ready to enter the post-high school job market, and compete in a global economy (Common Core State Standards Initiative, 2010). In 2010, the Arizona Department of Education (ADE) incorporated the Common Core State Standards into a new mathematics standards document for Arizona. The 2010 Arizona Mathematics Standards are nearly identical to the Common Core State Standards with a small number of additional mathematics concepts to fit the state's academic goals (Arizona Department of Education [ADE], 2011). Beginning in the 2011-2012 school year, the Chandler Unified School District mandated these mathematics standards to be taught to kindergarten, first grade, second grade, and seventh grade students, with implementation in third grade, fourth grade, fifth grade, sixth grade, eighth grade, and high school taking place incrementally throughout the 2013-2014 school year.

Chandler Unified School District has mapped out the teaching of the mathematics standards by quarter. Individual teachers or teams of teachers at the same worksite are responsible for ensuring that the standards are taught to mastery level. At my school the majority of the teachers plan their math instruction independently. But at some grade levels teams of teachers work together to plan instruction or one teacher does all of the planning for the grade level team. I am responsible for planning the second grade mathematics instruction with input from my team. We meet regularly to talk about how we perceive the successfulness of the mathematics instruction and learning, to look at student mathematics work, and to plan our next instructional steps. I research instructional techniques, materials, and resources and present them to my team. The team discusses them and decides if we will incorporate them into our instruction. If we agree, then I write the lesson plans. Our approach is collaborative and works well for ensuring adequate mathematics instruction is delivered to all San Marcos second graders. But even with this collaborative approach, instruction has not been effective at helping students retain or apply what they are taught in regards to problem solving.

Skills needed for successful mathematics problem solving at school begin developing in the preschool years when young children are given opportunities to manipulate the base ten number system (Montague, n. d.). The majority of San Marcos students come from disadvantaged homes–either financially, educationally, or both. Eighty-five percent of my students receive free or reduced price lunches and only 30% of my students have attended formal preschool. Because of these disadvantages, providing an adequate, engaging, and equal mathematical learning experience in the early elementary years is vital to increasing their future academic success. Payne (2005) describes two types of poverty, generational and situational. “Generational poverty is defined as being in poverty for two generations or longer. Situational poverty is a shorter time and is caused by circumstance (i.e., death, illness, divorce, etc.)” (Payne, 2005, p. 3). Situational poverty has always been an issue for the students attending San Marcos as many families have recently moved to government housing located near our school; but in the past three years, we have seen an influx of students coming from homes of generational poverty. With this change, many students are coming to school with a new set of issues; more students are demonstrating off-task behavior in the classroom, which I believe is due to a lack of motivation to learn in the formal school setting and a lack of the home support which leads to a well rounded educational experience (Payne, 2005). In addition to the socioeconomic disadvantages San Marcos students face, nearly half of my second graders speak English as their second language, so receiving assistance with homework and class assignments printed solely in English is problematic.

The accumulation of all of these factors has shown to be detrimental to San Marcos students. The Spring 2011 administration of the Arizona Instrument to Measure Standards (AIMS) found that only 56% of third graders, 61% of fourth graders, 52% of fifth graders, and 27% of sixth graders scored in the “meets” or “exceeds” categories on the mathematics subtest. This downward trajectory is confirmed in the finding that on the Spring 2010 AIMS mathematics subtest, 46% of fifth grade students met or exceeded the standard, but on the Spring 2011 assessment, for the same group of students then in the sixth grade, only 27% met the proficiency standard.

Mathematics problem solving skills are vital to all members of society. Perseverance, critical thinking, reasoning, planning, and justifying thinking are all skills students develop through carefully crafted and guided mathematics problem solving experiences (NCTM, 2000). To find success in careers as adults, students need to develop mathematical problem solving abilities in school (Kilpatrick & Swafford, 2002). In a recent newspaper interview, Jason Bagley, government affairs manager for Intel, the largest private employer in Chandler, stated that the main qualities his company looks for when hiring new employees is the ability to problem solve and be creative (“Intel Wants Cities”, 2010). It is important to me, my school, and my district that we counter the trend of low achievement in mathematics problem solving with an increased focus on best practices in mathematics. In addition to the much needed increase in AIMS test scores, we acknowledge the vast benefits of preparing our students for future endeavors, such as higher education, careers, and entrepreneurial experiences.

Previous interventions at the local level. To assist teachers at College and Career Readiness schools (formerly called Title I), as well as teachers at higher socio-economic schools, the Chandler Unified School District provides a variety of professional development supports for its teaching staff. A wide array of mathematical concepts and methods are taught to teachers through professional development courses. Additionally, the district has a Math Cadre, of which I am a member, that meets throughout the school year to create assessments, locate resources, design lessons that incorporate technology, and learn teaching methods and recent research results that can be shared with staff at their home schools. Instructional Resource Center staff members, who are district personnel, serve as resources and evaluate teachers' lessons, share strategies and teaching ideas, and act as coaches if requested. Additionally, because San Marcos is a College and Career Readiness School, the school receives additional monies that we use to fund a curriculum specialist. Our curriculum specialist is responsible for working alongside the principal in guiding and overseeing instruction, as well as presenting general teaching technique information and engaging the staff in professional development.

During the 2010-2011 school year, I was the researcher-practitioner on an action research project that involved investigating the effects of having more knowledgeable peers describe their solution strategies to Cognitively Guided Instruction-style (CGI) multiplication word problems to the class. The goal of this study was to increase students' problem solving abilities and solution strategy complexity. Results showed that students used a variety of strategies, such as direct modeling and derived facts, and the effect of the more knowledgeable peer sharing solution strategies was dependent on the type of multiplication word problem. In some cases the more knowledgeable peer had a positive influence on solution strategy complexity and in other cases it did not.

An additional major support that Chandler Unified School District began providing to select schools during the 2010-2011 school year was the involvement in a school improvement process called Data Wise. Data Wise is a cyclical process of examining testing data and classroom observation data to find a learner centered problem and a problem of practice that the school is facing. This research based process was created by the Harvard Graduate School of Education with research done in Boston Public Schools (Boudett, City, & Murnane, 2005). San Marcos was one of ten schools in the Chandler Unified School District that was selected to participate in this process. San Marcos found that mathematics problem solving was an area of weakness for its students. The committee contended that this was caused by the lack of mathematics problem solving experiences in which teachers engaged their students. Because of this, the Data Wise leadership team, of which I am the head, worked alongside San Marcos teaching staff to select the use of schematic representations as a school-wide focused teaching strategy that would help students improve their mathematics problem solving skills.

A schematic representation is a student created drawn or written depiction of a word problem that aids in the successful completion of the problem. Schematic representations vary from student to student, and can take the form of graphs, drawings of manipulatives, labeled sketches, or tables, among other things (Montague, n. d.). A schematic representation differs from a pictorial representation in that a schematic representation includes mathematically significant drawings and representations of the parts of the problem. For example, a schematic representation would show the spatial relationships among objects in the problem, could include numbers and labels integral to solving the problem, and aids in the solution of the problem. A pictorial representation would focus on the drawing of items in a problem, would not include accurate information that aids in finding the solution to the problem, and may contain extraneous drawings (Edens & Potter, 2008; van Garderen & Montague, 2003).

San Marcos teachers chose to focus on schematic representations because of favorable research about the problem solving strategy. NCTM (2000) states that students should be given multiple opportunities to produce schematic representations for problems to aid in the problem solving process. Others have found that problem solving ability increases as students visualize and then create a written schematic representation of the problem, whereas drawing a pictorial representation decreases problem solving correctness (Edens & Potter, 2008; Hegarty & Kozhevnikov, 1999; Van Essen & Hamaker, 1990; van Garderen & Montague, 2003). Foster (2007) found that her class of fourth grade students had a better understanding of the information provided by the problem and what the mathematics word problem was asking when her students created a schematic representation as part of their solution strategy. Little opposition has been shown toward the use of schematic representations at the elementary level. One hazard that Hegarty, Mayer, and Monk (1995) found was that students struggled more with making a useful written problem representation than actually computing the answer, which may have been due to students focusing on key words or dwelling on the pictures they were drawing rather than the mathematics behind the written representation (Presmeg, 1986). San Marcos teachers were aware of this hazard and took steps to counter it from the onset of implementation of the strategy.

To alter teachers' current mathematics instruction habits, teachers were directed to teach mathematics through problem solving a minimum of two times per week, give bi-weekly formative assessments and report students' scores to the Data Wise team, and create and administer quarterly summative problem solving assessments and report students' scores. School-wide improvement goals were set, with two main targets. First, grade levels aimed to increase the percent of students who scored proficient on quarterly problem solving assessments by 5% each quarter and to increase the total mathematics problem solving scores on the Spring 2011 AIMS assessment by 5%. At the end of the 2010-2011 school year, every grade level in the school surpassed their quarterly assessment goal. Results were not so overwhelmingly favorable for the AIMS assessment though. Third and fourth grade students narrowly met their AIMS problem solving goal, but fifth and sixth grade students fell short. Though the Data Wise process brought unity in instruction to the school and an increased awareness of the benefits of teaching mathematics through problem solving, it did not lead to the positive gains in AIMS mathematics test scores that it set out for. All of this led me to some essential questions. Why were these stagnant test scores continuing to occur? What happens if students lack the background experiences and knowledge needed for mental imagery to create schematic representations of problem situations?

Many different attempts to remedy the problem of low achievement in mathematics problem solving have been made, and even with all of the resources and interventions San Marcos Elementary and the Chandler Unified School District had in place, my second graders were still struggling to solve complex beginning second grade level word problems. To address this issue, in this study I investigated:

1.) How does a class of second grade students at San Marcos Elementary solve Cognitively Guided Instruction-style contextual word problems?

2.) How and to what extent does partnered Cognitively Guided Instruction-style mathematics word problem solving through guided incremental steps affect a class of San Marcos second graders' mathematics problem solving abilities?

CHAPTER 2

REVIEW OF SUPPORTING SCHOLARSHIP

Historical Views of Problem Solving Instruction

Problem solving in elementary grades mathematics is defined primarily as drawing on knowledge, skills, and experiences to engage in a task for which the solution method is unknown (NCTM, 2000). Throughout most of the nineteenth and early twentieth centuries, the traditional educational view of problem solving was for a teacher to teach a mathematical concept or algorithm to the class through direction transmission, give the students multiple rote exercises to complete to practice the skill, and then, if time permitted, assign word problems that required students to apply the algorithm (D'Ambrosio, 2003; Mickelson & Ju, 2011). This format of simply applying a known algorithm to a problem in context does not fit NCTM's current day definition of problem solving. Authentic problem solving requires that the solution be unknown and the student must do more than simply insert numbers from the problem in a given algorithm (NCTM, 2009). This traditional format of textbook problem solving instruction has been found to be unsuccessful in improving the learning of students at risk of mathematical difficulties (Jitendra et al., 2007).

In kindergarten through second grade, mathematics problem solving has historically been in the context of the arithmetic operations with whole numbers: join (addition), separate (subtraction), part-part-whole (addition and subtraction), compare (subtraction), grouping (multiplication), and portioning (division) problems. More recently it has begun to encompass all subsets of mathematics education, including problems that involve more than just addition, subtraction, multiplication, or division, such as fractional equivalencies and geometric properties (Carpenter et al., 1999).

Pólya's (1957) problem solving hierarchy has formed the basis for much of the classical teaching of mathematics problem solving. Pólya describes the process as understanding the problem, devising a plan, carrying out the plan, and reviewing solutions. Strategies for solving problems suggested by Pólya are vast, including guess and check, solve a simpler problem, act it out, draw a picture or diagram, work backwards, and look for a pattern (Pólya, 1957).

Problem solving as a way to teach mathematical concepts began in the late 1970s and has gathered strength since then (Schoenfeld, 1992), but this trend has not been without its critics. Stacey (2005) argues that teaching mathematics through problem solving is not a best practice. Problem solving should be seen as the goal of mathematics, rather than the method. Avital and Barbeau (1991) posit that students may come to wrong conjectures and conclusions if they are encouraged to solve problems based solely on intuition. Indeed, a balance between intuition, reasoning, teacher guidance, and formal instruction need to be present at various times when children are working to develop their understanding in a mathematics classroom. Throughout this time the NCTM has held firm to its belief that problem solving is a key component of, and the primary reason for, learning mathematics. NCTM explains that learning to solve problems is the major goal of mathematics instruction in its three foundational publications, *Agenda for Action* (NCTM, 1980), *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), and *Principles and Standards for School Mathematics* (NCTM, 2000).

Most recently, the Standards for Mathematical Practice have been introduced to facilitate the proper implementation of the problem solving standards contained in the Common Core State Standards into the mathematics classroom (Common Core State Standards Initiative, 2010). These standards align with NCTM's Process Standards, which have guided mathematics instruction in the past. Standards for Mathematical Practice, as they relate to problem solving at the second grade level include having students: persevere to solve problems; reason in their head abstractly; share and discuss strategies; represent and model problems; use tools, pictures, drawings, manipulatives, and objects appropriately to aid in problem solving; check if solutions make sense and are correct; use prior mathematics knowledge in novel situations; and look for similarity among problems to assist in solving current problems. These Standards for Mathematical Practice were designed to help students focus on the process of solving problems, and improve their holistic problem solving skills, rather than just focusing on coming to a correct solution rapidly and possibly without understanding (White & Dauksas, 2012).

The Effect of Poverty on Problem Solving

Among the world's wealthy nations, the greatest disparity between the rich and the poor occurs in the United States (Wilkinson & Pickett, 2009). In recent years, there has been an increase in the percentage of U.S. schools with 75% or more of their population receiving free or reduced price lunches, which is commonly accepted as an indication of socioeconomic status of a neighborhood (Lee, Grigg, & Dion, 2007). During the 1999-2000 school year, 12% of U.S. schools fit into this category of high percentage of free or reduced price lunches, but in the 2007-2008 school year, the amount had jumped to 17% (Aud et al., 2010). Gonzales et al. (2008) found that when a school had more than 75% of its students in poverty, the mathematics score on the 2007 Trends in International Mathematics and Science Study was a whole standard deviation lower than highly affluent schools. From these testing results it is easy to see that poverty and opportunities provided by wealth greatly influence test scores and student achievement (Berliner, 2011).

Students from lower socioeconomic backgrounds generally enter school with less language exposure, limited vocabulary range, and less background knowledge, which may be due to less exposure to printed text in the preschool years (Coley, 2002; Ramey & Ramey, 1994; Raver & Knitzer, 2002; Senechal & Cornell, 1993). Research has shown that children from very low socioeconomic status homes were read to on a daily basis much less frequently than children from very high socioeconomic status homes, 34% as compared to 63% (Coley, 2002). This limited language experience typically relates to lower problem solving abilities (Coley, 2002; Guerra & Schutz, 2001). Socioeconomic status is also directly related to approaches to learning, with higher levels of socioeconomic status relating to higher engagement, persistence, and on-task behavior (Yair, 2000; Marks, 2000). Cognitive development occurs faster in students who are actively engaged in their learning; therefore, mentally active children have faster cognitive development than passive children (Kamii & Rummelsburg, 2008). This coincides with the notion that greater levels of participation, attention, and task persistence are correlated with higher standardized test scores and higher performance ratings from teachers (Alexander, Entwisle, & Dauber, 1993; Duncan et al., 2007; Finn, Pannozzo, & Voelkl, 1995; Horn & Packard, 1985; McClelland, Morrison, & Holmes, 2000; Schaefer & McDermott, 1999; Tramontana, Hooper, & Selzer, 1988; Yen, Konold,

& McDermott, 2004). Bodovski and Youn (2011) found that higher ratings of students' approaches to learning–persistence, on task behavior, engagement–in first grade were substantially related to higher reading and mathematics achievement of these students when they were in fifth grade.

A teacher can help a child develop the background knowledge needed to solve mathematics word problems by teaching vocabulary and providing experiences to which the child can relate future knowledge (Kovarik, 2010). Because students enter school with extremely varied background experiences and skills, the best curricular decision a teacher can make is the developmentally appropriate decision for each child. Providing engaging and interesting experiences daily that individualize the learning process for each student is imperative for school success (Ramey & Ramey, 1994). Poor minority students are generally not afforded this luxury. Their mathematics instruction typically focuses on memorizing facts and information and test taking tips, which increases even more as the schools these students are enrolled in do not make adequate yearly progress. Students are less likely to be given the opportunity to problem solve in mathematics, develop their critical thinking skills, or write creatively (Berliner, 2011). Because socioeconomically disadvantaged and minority students are not given opportunities to problem solve or use logic to reason through problems, their skills will generally not increase.

Theoretical Views Influencing Problem Solving Instruction

A theoretical approach that combines Vygotsky's social development theory, Bandura's social learning theory, and Piaget's and Vygotsky's theories of constructivism address the necessary attributes for a successful primary mathematics classroom. A primary component of social development theory is the zone of proximal development, which is “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86). By interacting with a more capable peer or a teacher, commonly known as the MKO (more knowledgeable other), students’ cognitive abilities can be developed to the point where a task that had been too difficult to complete without support, can now be accomplished independently (Vygotsky, 1978). Cloutier and Goldschmid (1978) found that elementary students who were given the opportunity to talk about their mathematics solution strategy with a classmate showed considerable ability improvements, but peers who completed the same tasks without being allowed to discuss their work with a partner did not improve their skills at all.

Social learning theory purports that children in classroom situations learn from each other through observing each other’s action, observing the outcomes of that action, and deciding if the observer will replicate the action (Bandura, 1977). In an elementary mathematics classroom, students are involved in the processes of these theories when learning mathematics problem solving. Students working in small groups observe each other dissecting, understanding, computing, and reasoning through problem situations posed by the teacher. When students are shown other ways to solve problems at a level just above where they are currently cognitively functioning, their mathematics abilities are developed (Carpenter et al., 1999). Carefully constructed learning activities in the form of repeated, similar problems will allow the child to practice what he has observed and continue to develop and solidify his skills and understanding. As explained by social development theory and social learning theory, students learn from each other in a way that cannot be taught solely through direct transmission from a teacher. These theories would support the idea that a good teacher can be defined as “one who gets students to explain things so well that they can be understood” (Reinhart, 2000, p. 478) rather than “one who explains things so well that students understand” (Reinhardt, 2000, p. 478).

Vygotsky’s (1978) idea that young children’s mathematical learnings begin before they attend formal schooling directly coincides with what Carpenter et al. (1999) use as the basis of their theory of Cognitively Guided Instruction. A teacher can create a classroom environment that leads to social constructivism, the belief that students learn best in group settings and that relating school learning to real world situations will help students build their mathematical understandings (Vygotsky, 1978). NCTM (2000) also suggests that in elementary mathematics classrooms, teachers should employ constructivist-style practices and tools to engage students in creating their own meaning based on their preexisting knowledge since the constructivist view coincides with how students’ brains learn mathematics (Zambo & Zambo, 2007). Piaget (1953) and Vygotsky (1962) propose similar views of constructivism in the classroom. Piaget’s cognitive constructivism, specifically logico-mathematical knowledge, suggests that ideas are constructed by individuals based on their interpretation of situations (Kamii & Rummelsburg, 2008; Kamii, 2012), whereas Vygotsky’s social constructivism proposes that ideas and understandings are created through interactions with peers and teachers. An effective mathematics classroom employs both practices at varying times, with the teacher facilitating learning experiences based on students’ current level of understanding. Mathematical problems should be posed and discussions should take place that help the child build on what the child already knows (Harel & Behr, 1991; National Research Council, 1989; Powell & Kalina, 2009; Romberg & Carpenter, 1986). Problems that are non-routine and cause disequilibrium require students to relate what they know, their schemata, to the problem presented to them. Learning through problem solving helps students develop deep understanding of mathematical concepts by broadening the students' prior knowledge through the induction of new understandings (Lambdin, 2003). Staub & Stern (2002) found a positive correlation between the classroom achievement gains on word problems and teachers' cognitive constructivist orientation in their longitudinal study of 496 German elementary students. They also found no negative impact of teachers' higher level of cognitive constructivist orientation and students' arithmetic skills, defying what some believe is a drawback of the constructivist theory of mathematics education.

Current Trends in Problem Solving Instruction

As high-stakes testing and implementation of the Common Core State Standards have increased the demand for student problem solving abilities, teachers are feeling the strain on their class time. Teachers are often directed to teach mathematics in a more conceptual or engaging way, but are rarely given meaningful suggestions on how to do that. Daro, one of the primary authors of the Common Core State Standards, admits that the Standards do not dictate how the content should be taught (Daro, 2011). Teachers are adapting well-known strategies and creating new strategies to meet the needs of students while meeting the teachers' needs for increased student achievement without dramatically increasing teaching time. For example, two recent studies, Wu et al. (2009) and Zollman (2009), have incorporated the use of graphic organizers to assist students with the comprehension, evaluation, and strategy of solving word problems. Wu et al. (2009) found that using graphic organizers in conjunction with the mathematician's chair, a protocol for sharing mathematics work and thought processes, increased a group of high performing second graders' California Standards Test scores by 3.7%. Zollman (2009) found an increase of 42% in the problem solving abilities of a group of third through fifth grade students on open-response mathematical problems after applying the graphic organizers innovation.

Recently, great emphasis has been placed on the classroom climate and culture. Research has shown that it is necessary for teachers to provide safe, rigorous, mathematically rich environments for students to develop their mathematics problem solving abilities (Sutton & Krueger, 2002). Collaboration, discussion, and justification are keystones for an effective problem solving lesson. Classrooms where children share their reasoning and engage in discussions better prepare students of all backgrounds for advanced mathematics (Chevalier, Pippen, & Stevens, 2008). Additionally, students' enthusiasm for mathematics increases when given the opportunity to work on problems in a fear-free and pleasurable environment. Students feel more liberated to take risks and errors are not viewed as failures, but as opportunities for personal growth and learning (Femiano, 2003).

In addition to climate and culture, the mathematical word problems that the teacher selects has gained recent focus and is now realized to have great importance to developing mathematical problem solving skills. Because increased time is spent on problem solving, problems must be formulated that facilitate children learning mathematical concepts. Problems that are open-ended (meaning that there are many ways to access the solution), problems that are content developing, and work designed as an anticipatory set are beneficial to elementary students (Hartweg & Heisler, 2007). With proper problem selection, teachers can guide students toward the goals of developing a broad range of problem solving skills and strategies and monitoring and reflecting on their mathematical thinking and work (NCTM, 2000).

Even with our advances in the understanding of mathematics teaching and learning, teachers are still faced with curriculum materials dictated by districts that do not align with research showing the need to create culture, foster student discourse and discussion, and build collegiality in the mathematics classroom. Teachers have found that they must adapt their mandated curriculum to fit their students' needs and their beliefs about teaching (Drake, Cirillo, & Herbel-Eisenmann, 2009). Recent research has supported the use of Cognitively Guided Instruction (CGI) for guiding students in their mathematical development. This has led to some schools and teachers adopting CGI in their math instruction.

CGI is a framework that was designed to assist elementary school educators, specifically those teaching kindergarten through third grade, to understand and help develop their students' problem solving abilities. A main tenant of CGI is that children enter school with innate abilities and knowledge about mathematics. Children use these abilities to construct solutions to problems that many adults would consider far beyond a child's mathematical abilities. Through problem solving experiences using manipulatives, thinking about their own solutions, and learning other students' solutions, children's mathematical prowess develops through a series of steps, beginning with modeling problems and ending with learning number facts to efficiently solve problems (Carpenter et al., 1999).

CGI posits that there are 11 different types of addition and subtraction word problems and three multiplication and division word problem types, all dealing with single digit and multi-digit numbers (Carpenter et al., 1999). Figure 1 shows the 11 different addition and subtraction problem types with sample problems given as examples. Since children bring a vast informal system of strategies to solve these problems with them when they come to school, it is the teacher's duty to provide problems with which children can grapple, fostering children's movement across the continuum of sophistication of solution methods (Wisconsin Center for Education Research, 2007). Teachers facilitate the development of students' understanding of mathematics by allowing them to use manipulatives, draw pictures, discuss their solutions, listen to other students discuss their problem solving strategies and solutions, and build connections among problems (Carpenter et al., 1999). Because algorithms and computations are not the primary focus of CGI, students do not feel restricted in their problem solving strategies. Students are encouraged to solve problems however they are able, and by watching and listening to other students solve the same problems, students' solution strategies become more advanced, developing from direct models, to counting strategies, to derived number facts (Carpenter, Fennema, & Franke, 1996; Carpenter et al., 1999).

Join	*(Result Unknown)* Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?	*(Change Unknown)* Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	*(Start Unknown)* Connie had some marbles. Juan gave her five more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	*(Result Unknown)* Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	*(Change Unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	*(Start Unknown)* Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-Part-Whole	*(Whole Unknown)* Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	*(Part Unknown)* Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?	
Compare	*(Difference Unknown)* Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	*(Referent Unknown)* Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Figure 1. CGI addition and subtraction problem types with examples. Problem types assessed in this study are shaded in gray. Reprinted from *Children's Mathematics: Cognitively Guided Instruction* (p. 12), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright 1999 by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson.

Problem solving solution strategies generally follow a hierarchy of complexity, with some solution strategies lending themselves better to certain problem types. Children generally begin solving problems by using tangible items, such as cubes or their fingers, to model the action or relationship stated in the problem. This is referred to as Direct Modeling (Carpenter et al., 1999). Jordan, Kaplan, Ramineni, and Locuniak (2008) found that students generally use their fingers to model number sentences when they are first learning number combinations, but tend to lessen the use of this strategy as their understandings of number relationships mature. This generally happens sooner for higher socioeconomic students, usually by beginning of second grade; but for lower socioeconomic students the developmental process is slower (Jordan et al., 2008). Students then progress to using Counting strategies that show a child understands that it is not necessary to model and count each set in the problem. Students employing a Counting solution strategy may still use manipulatives, but the manipulatives are now used to keep track of the numbers they are counting, not to represent individual numbers in the problem. As the student's understanding of the operations and the relationships between numbers increases, they begin to rely less on modeling and counting to find answers and begin to use Derived Facts and Recall of Number Facts. A Derived Fact is a fact, not memorized, that students arrive at based on a memorized fact and their understanding of number and operation (Carpenter et al., 1999; Kling, 2011). Figure 2 illustrates the solution type complexity hierarchy for addition and subtraction problem types. Children also tend to use certain solution strategy subtypes while they are in the Direct Modeling strategies and Counting strategies stages.

Complexity Level	Solution Strategy	Description	Solution Strategy Subsets
Level 1: Most basic	Direct Modeling	Using manipulatives (counters, unifix cubes, fingers, etc.) to model a problem and represent each number. Then counting the manipulatives to find the answer.	Joining All Joining To Separating From Separating To Matching Trial and Error
Level 2: Intermediate complexity	Counting	Counting on or counting back from a given number to find the solution.	Counting On From First Counting On From Larger Counting On To Counting Down Counting Down To
Level 3: Most advanced	Number Facts	Using known addition or subtraction facts to solve problems or to aid in the solution of a problem containing a number set for which the solution is not memorized.	Derived Facts Recalled Facts

Figure 2. Solution strategy complexity hierarchy. Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 15 - 30), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright 1999 by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson.

Coinciding with the problem solving strategy stages of CGI is the notion that problem solving skill develops through a concrete to pictorial to abstract format (Piaget, 1953). During the early 2000s, problem solving work based on this principle gained momentum with the increased interest of the Singapore model (Englard, 2010) and concrete-representational-abstract (CRA) (Flores, 2010). The Singapore model of problem solving poses that after students use real life objects to model the situation in a word problem, they are ready to draw a pictorial representation that shows the actions of the problem. This helps students understand the relationship of the words of the problem and the mathematical operations involved in its solution. In an experiment that compared a class of third grade students who received instruction using the Singapore model method to third, fourth, and fifth grade classes that did not receive the targeted instruction, the class of third graders who received instruction out performed all other classes on a test of word problem solving (Englard, 2010). CRA has widely been used with students who are at risk of failure in mathematics, as well as students with learning disabilities in mathematics. CRA's teaching and learning process, that develops from concrete to representational to abstract, begins with the use of manipulatives. The teacher models the proper use of manipulatives to solve word problems, and then guides the students in using the manipulatives to solve problems independently. From there, manipulatives are replaced with pictures or drawn models representing the problem solving process. Finally, students transition to the abstract phase, where they may use mnemonic strategies to help remember how to solve a problem fluently using numbers (Flores, 2010). Sometimes, the concrete stage entails more than just the typical mathematics manipulatives of unifix cubes, color tiles, and base ten blocks. Arzarello, Robutti, and Bazzini (2005) used artifacts, commonly called realia, and full body movements to model story problems through actions. The effectiveness of this acting out strategy was tested through a teaching experiment with eleven- and twelve-year-olds. The basis of the experiment was that meaning can be constructed through one's real experiences. The study showed that students were actively engaged in creating a physical model of the story problem and this fostered understanding when students moved on to the abstract phase (Arzarello et al., 2005). Strand (1990) also found her students successful when she employed acting out story problems as the first strategy her students used when finding word problem solutions. During this phase, Strand also wrote the equation that corresponded to the acted out model on the whiteboard to scaffold her students. This acting out process was used to solve addition, subtraction, division, and multi-step word problems, and then students were transitioned to using manipulatives, such as base ten blocks (Strand, 1990). Visually observing objects and graphically representing them, either schematically or numerically, are skills that positively impact students' problem solving abilities (Cuoco & Curcio, 2001).

By doing word problems with the whole body, connections are made between the movements needed to solve the mathematics word problem, visualization of the memory of the actions needed to solve the problem, and the cognitive processes used to solve the problem. Mickelson and Ju (2011) used bodily movements termed math propulsions, the acting out of problems, and social interactions to increase the conceptual exploration and understanding of their secondary school mathematics students. Through math propulsions, students had direct physical control over mathematics variables, used vocabulary and discussion to immediately act on a math problem, and saw math as an open ended inquiry rather than a domain regulated by lock-step routines to reach one correct solution (Mickelson & Ju, 2011).

When confronted with a mathematical translation of a real-world problem, primary aged students commonly arbitrarily or randomly combine the numbers in the word problem; this process of number grabbing is commonly due to difficulties comprehending the text and how the situation fits into the mathematics problem solving realm (Peter-Koop, 2005). Good teaching should build on students' existing knowledge and understandings, which form the basis for new learning (Romberg & Carpenter, 1986). Solving problems logically and intuitively leads to a deep, rich understanding of mathematics that can be applied in novel situations (NCTM, 2000; Van de Walle, 2004), but the connection between the real world and the mathematical world does not happen intuitively for some students (Onslow, 1991). Because many socioeconomically disadvantaged students lack background experiences to immediately "see" the steps needed to solve a word problem, teachers have guided students to problem solving success by starting at the visualization level.

Hegarty et al. (1995) found that effective problem solvers unpack the text of a problem by translating each sentence or action in the problem into a visual representation in a mental model of the problem, sometimes called seeing in the mind's eye. Using the mind's eye, mentally recalling pictures of objects or events not currently visible with the eye (Block, 1981; Edens & Potter, 2008; Hibbing & Rankin-Erickson, 2003; Kosslyn, Pinker, Smith, & Shwartz, 1981; LeBoutilier & Marks, 2003; Paivio, 1971, 1983, 1986; Sadoski & Paivio, 2001), can aid in various types of problem solving, but most students must be taught how to create these mental images (Douville, 2004). Visualizing using the mind's eye has been shown to be beneficial in mathematics, science, architecture, engineering, and technology education (Kaufman, 2007).

The Sensory Activation Model (SAM), a visualization technique, was used with second grade students in the contexts of reading and writing. Over a six day period, teachers modeled how to create rich mental images that included all senses and then allowed students to create their own SAM images. This procedure led to increased description in written and oral responses than before the SAM procedure was employed. Students also carried SAM over to mathematics work (Douville & Boone, 2003). SAM can be used as a bridge to lead students from the concrete world of the prior experiences they bring with them to the classroom to the abstract representations needed to efficiently and effectively solve mathematics word problems (Douville, 2004). Douville (2004) suggests that pictorial image drawing must first be internally imagined before the student can draw it. Teachers should be cautious as to not expect that students can intuitively use mental imagery. Teachers may need to provide background knowledge and vocabulary that relates to mental images while modeling how to create images in the mind's eye (Hibbing & Rankin-Erickson, 2003). It is important for teachers to help young students make connections between their background knowledge and authentic mathematical tasks to help concretize the process (Carpenter et al., 1999; Douville, 2004; NCTM, 2000). Visual maturity is generally attained between ages 8 and 11, though many students do not automatically develop the ability to visualize a mathematics word problem. These students will need instruction on how to visualize to represent problems (Montague, n. d.). To help transition from the beginning stage of problem solving development to the representational stage, it may be necessary for teachers to model how this transition takes place. The innovation that I implemented was designed to help build this bridge in a developmentally appropriate format.

CHAPTER 3

RESEARCH DESIGN

Mixed Methods Purpose and Design

This study was designed to investigate how a class of second grade students solved mathematics word problems in order to increase students' problem solving skills that may transfer to other academic subjects and domains in their lives. The focus was to determine the effect of using a sequence of representations to solve word problems (the independent variable) on students' scores on pre-/post-assessments and daily problem solving (dependent variables). This practical action research study employed mixed methods to gain a deeper understanding of the findings than a qualitative or quantitative study could produce on their own. The results of this study will be used to influence the day-to-day activities of my classroom (Caracelli & Greene, 1993; Creswell & Plano Clark, 2007; Greene, Caracelli, & Graham, 1989; Stringer, 2007; Tashakkori & Creswell, 2007; Teddlie & Tashakkori, 2006; Woolley, 2009).

This study employed a one-group pre-test-post-test design, that is a pre-test, then a treatment, followed by a post-test (Creswell, 2009). Quantitative data included scores on pre-/post- problem solving assessments, analysis of solution strategies used on the pre-/post-assessment, answers to students' daily problem solving tasks, length of time students solved daily word problems, and the number of words students spoke while solving daily problems. Figure 3 shows the research design for the collection of pre-/post-assessment quantitative data in this study.

Group A O1 ------------- X --------------O2

Figure 3. Implementation of the one-group pre-test-post-test design. O1 indicates the pre-test phase, X indicates the implementation of the innovation, and O2 indicates the post-test phase. Adapted from *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (p. 160), by J. W. Creswell, 2009, New Delhi, India: Sage. Copyright 2009 by Sage Publications.

Qualitative data analysis was based on grounded theory, which allowed the qualitative data to be used to emit complex understanding of the situated context (Creswell, 2009). Qualitative data analysis took place over a series of steps throughout the study. Data from daily video recorded observations and students' explanations of their solution strategies on the pre-assessment and post-assessment were analyzed using the open coding, axial coding, and selective coding format (Corbin & Strauss, 2008).

Throughout this study, quantitative and qualitative data were gathered concurrently using a component design (Greene, 2007). Quantitative data and qualitative data remained identifiable throughout the data collection process, although because triangulation was employed, both quantitative and qualitative methods were necessary to create final assertions (Creswell, 2009; Greene, 2007). Figure 4 shows the triangulation design used in this study. Triangulation allowed multiple data sources–the correctness of student answers to daily contextual problems, written descriptions of students' problem solving strategies on the pre-assessment and post-assessment, weekly video recorded observations of three dyads' solution processes, and solution strategies and sub-strategies on the pre-assessment and the post-assessment–to be mixed at the analysis stage and to weigh in on the findings of this study (Creswell, 2009; Greene, 2007). Through triangulation, convergence, not divergence, was elicited.

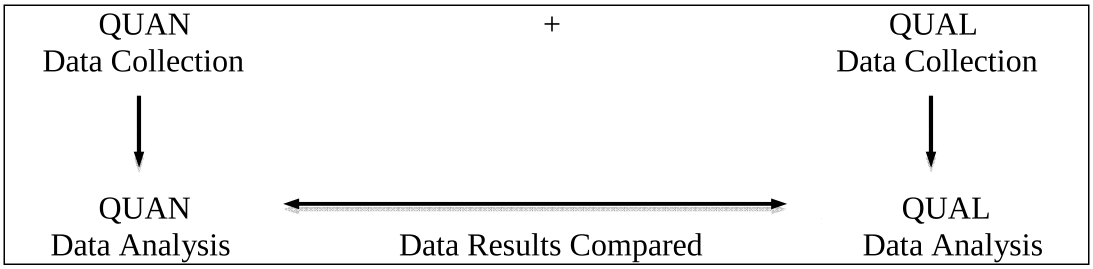

Figure 4. Concurrent triangulation design of this study. Quantitative and qualitative data were collected concurrently and were brought together at the analysis phase. Reprinted from *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (p. 210), by J. W. Creswell, 2009, New Delhi, India: Sage. Copyright 2009 by Sage Publications.

Setting and Participants

This study took place at San Marcos Elementary, a suburban College and Career Readiness school that services preschool through sixth grade students. In the 2012-2013 school year, San Marcos had a population of 635 students from six different racial and ethnic backgrounds (83% Hispanic, 6% White, 6% African American, 2% Native American, 2% Asian, and 1% other races). Ninety-one percent of the students received free or reduced price lunches. Each year, nearly two-thirds of our students enter kindergarten as English language learning (ELL) students. Historically, our ELL reclassification rate has been approximately 40% each year. This means that many students who participated in this study were classified as fluent English proficient (FEP). FEP students are students who have tested out of the English language development program within the past two years, but are still developing their full English language skills (Liquanti, 1999). San Marcos Elementary did not make Adequate Year Progress (AYP), as dictated by the No Child Left Behind Act of 2001, during the 2009-2010 or 2010-2011 school year, but did make AYP during the 2011-2012 school year, and continues to undergo shifts in pedagogical practices.

The participants in this study were my Fall 2012 second grade class of children. Nineteen students, ranging in age from 6 to 8 years old, participated in the entire innovation process. Students were assigned to second grade classrooms using a stratified random distribution. To place students in second grade classrooms, first grade teachers rated their students on academic ability and study skills. They then divided students so that each second grade classroom had an equal number of academically advanced students, academically typical students, and academically delayed students. This procedure was then crosschecked to ensure even distribution of students with strong study skills and weak study skills. Therefore, my class was comprised of students at all levels of mathematics achievement. This reduced the regression threat to internal validity (Smith & Glass, 1987). All students present in my regular education mathematics classroom at the time of each lesson participated in that lesson. Students in my homeroom who had an Individualized Education Plan (IEP) in the area of mathematics did not receive the treatment, and therefore were not included in the study. Those students received accommodated or alternative curriculum in the special education resource classroom. In this study, mortality–or participants leaving the study–was a limited threat to validity, with only three students who started the innovation changing schools during the implementation period (Smith & Glass, 1987). These students' data were not included in the study.

Innovation

This study used an adaptation to the traditional CGI format with students working in like-ability dyads to solve daily word problems and then sharing their solution strategies with the class. Students were guided through a seven phase solution strategy plan that scaffolded their problem solving skills, in hopes of increasing their abilities and efficiency. Guided problem solving began with using realia to act out the word problem. Students then moved on to using traditional mathematics manipulatives to model the problem. In the next phase, students drew schematic representations to solve the problem. Finally, students were guided to use number sentences to come to a solution. The innovation included a scaffolded hybrid period between each of these phases that was designed to assist students in their transition from one problem solving solution strategy to the next.

Justification for the innovation. CGI was chosen because of its direct impact on the formation of the problem solving standards of the Common Core State Standards, which now guide my mathematics instruction (Dacey & Polly, 2012). This daily problem solving format addressed all aspects of the Standards for Mathematical Practice as stated in the Common Core State Standards, the benchmark for problem solving in the mathematics classroom (White & Dauksas, 2012).

Pre-assessment. Each student completed a five question problem solving pre-assessment (Appendix A). The teacher, myself, who was also the researcher, delivered this assessment orally one-on-one. I sat in close proximity at a 90 degree angle to the student, across the corner of a classroom table. This position allowed me to see the student's hand and head movements, hear the student even if the child was speaking in a whisper tone, and see the child's manipulation of materials, all while being at a nonthreatening distance from the child. The location of the teacher is important to this study because oftentimes students appear to be using one solution strategy, such as Direct Modeling, but are actually using a Counting strategy (Carpenter et al., 1999). Being at a close proximity to the student gave me a better opportunity to observe the child's exact problem solving strategy. On the table there was realia (the actual object stated in the word problem), traditional mathematics manipulatives (base ten blocks and unifix cubes), paper, and a pencil. The student was instructed to use as many or as few of these materials as desired to solve each problem. Each student's solution strategy and strategy subset for each problem was recorded on the Solution Strategy Recording Form (Appendix B). If necessary, I asked the student to describe the solution strategy used to solve the problem. The pre-assessment took between 9 and 15 minutes per child to conduct.

Placement of students into dyads. Upon completion of the pre-assessment sessions, I analyzed the students' solution strategies and solution strategy subsets, and placed students in like-ability partner groups. During the entire innovation period, these like-ability dyads worked together to complete the daily CGI-style mathematics word problems.

Justification for like-ability dyads. The benefits of working in dyads are many, and Marzano (2007) recommends that students be placed in pairs or triads for cooperative group work. Dyads allow discussion between students that can increase students' engagement and persistence when solving a problem, as well as help lower ability students understand the mathematical meaning of a problem, its vocabulary, and the appropriate mathematical response to the problem (NCTM, 2004). In a study of fifth grade students solving an involved mathematics problem, students working in dyads explored multiple solution strategy paths and showed divergent reasoning that would likely not have been reached in individual problem solving (Vye, Goldman, Voss, Hmelo, & Williams, 1997). Schmitz and Winskel (2008) found that upper elementary students working in closer related ability groups performed better than dyads that contained one high performing student and one low performing student, and Denessen, Veenman, Dobbelsteen, and Van Schilt (2008) discovered that when sixth grade students were placed in mixed ability dyads, the higher ability student performed better on the problem solving task and showed more cognitive elaborations than the lower ability partner. Similar findings on like-ability groups have been found at lower elementary grades, as well. Takako (2010) suggests that when early elementary students at low socioeconomic schools participated in mixed ability groups, their scores in reading did not improve. Working in dyads also benefits at-risk students. Students typically reluctant to share ideas with the class usually feel more comfortable sharing the answer or solution strategy of a partner or small group (Reinhart, 2000).

Phases of the innovation. This innovation has seven phases, each lasting from one to three weeks. The seven phases progressed with increasing levels of mathematics complexity in their solution strategies: acting out the problem using realia, modeling the problem using traditional mathematics manipulatives, drawing a schematic representation of the problem on paper, and finally using a number sentence to solve the problem. Figure 5 displays the phases of the innovation process.

Phase 1	Phase 2	Phase 3	Phase 4	Phase 5	Phase 6	Phase 7
2 weeks	1 week	2 weeks	1 week	2 weeks	1 week	3 weeks
Acting the problem out using realia	Hybrid of Phase 1 and Phase 3	Using traditional mathematics manipulatives	Hybrid of Phase 3 and Phase 5	Drawing a schematic representation on paper	Hybrid of Phase 5 and Phase 7	Writing a number sentence on paper

Figure 5. Diagram of innovation phases implementation.

Justification for innovation phases. Problem solving phases were sequenced in this order because many students benefit from seeing the transition from manipulatives to schematic representations to written symbolic notation of problem solving processes modeled by the teacher or other more knowledgeable others (Montague, n. d.). Additionally, teachers should provide learning experiences and classroom discussions that foster the growth of the understanding that mathematical symbolism represents real world experiences and vice versa. Being a flexible problem solver, being able to use a variety of strategies to solve problems, saves time and effort. It is when people can travel from the concrete to the abstract and from the abstract to the concrete that they become mathematically literate (Onslow, 1991).

Phase 1, realia (2 weeks). In Phase 1 of the innovation process, students solved problems by modeling the actions in the daily CGI-style word problem using realia. Examples of realia are the actual items, or cutout models of the actual items, that are a component of the word problem's context. For the problem "Robin had 8 toy cars. Her parents gave her some more toy cars for her birthday. Then she had 13 toy cars. How many toy cars did her parents give her?" (Carpenter et al., 1999, p. 21), students would be given toy cars to model the actions of the problem or students could use cutouts of toy cars to act out the problem and find the solution. There was a risk during this phase that students might focus on playing with the realia rather than on the mathematics at hand. Luckily, Chevalier et al. (2008) found that even though they expected students to be more focused on mathematics realia (including play money, plastic cookies, rubber snakes, and cotton ball mice) than on the problem solving process itself, students ended up being focused on the mathematical concepts being taught. This was also the case in this study.

Justification for problem solving using realia. The realia phase may be critical to the successful completion of word problems, since many students have troubles solving word problems because they do not know where to begin and because they may not make the connection that a more abstract model, for example a wooden block, can represent the problem at hand (Montague, n. d.). The most common ways of teaching addition and subtraction word problems is through creating a number sentence and focusing on the solution strategy. Instead, instruction should begin with representing the situation in the problem, which is especially true for more difficult problem types (Willis & Fuson, 1988). Nuthall (1999) found that visual instruction–helping students generate mental pictures–and dramatic instruction–dramatizing content–both enhance learning and lead to increased retention. Nonlinguistic representations, in the form of mental images based on one's experiences, can be effective ways to process information (Marzano, 2007). In the case of this innovation, acting out the problem can lead to mental imagery that may be used as nonlinguistic representations on future problems.

Phase 2, connecting realia to traditional mathematics manipulatives (1 week). During this phase students used a hybrid solution strategy by modeling the problem's actions using realia first, and then using traditional mathematics manipulatives, a more abstract model, to represent the problem. Traditional mathematics manipulatives, such as base ten blocks, can be quite abstract and therefore should be used alongside the real world experiences they represent to build meaning behind the manipulatives (Onslow, 1991).

Phase 3, traditional mathematics manipulatives (2 weeks). During Phase 3, students used traditional mathematics manipulatives (base ten blocks, unifix cubes, or counters) to model the daily word problem and find the answer. These traditional manipulatives took the place of the realia used in the first phase of the innovation. Throughout this phase, implementation was focused on the idea that manipulatives do not guarantee engagement in the classroom. It is what students do with the manipulatives that evoke learning and understanding (Onslow, 1991).

Justification for problem solving using traditional mathematics manipulatives. This innovation was designed to scaffold students through the problem solving hierarchy to efficiency. Phase 1 was designed to use total body acting out experiences using realia to help students develop their mental imagery skills. As stated earlier, one of the most powerful problem representation strategies is visualization of the problem. Visualization can be in the form of mental imagery, manipulatives, or paper and pencil representations (Montague, n.d.). Phase 3 used traditional mathematics manipulatives to build on the previous phases, hence developing visualization of a posed mathematics problem.

Phase 4, connecting manipulatives to schematic representations (1 week). In this hybrid phase of the innovation students first solved the problem using traditional mathematics manipulatives and then drew a schematic representation of the problem. Since students who have difficulties representing problems will likely have troubles solving them, teachers need to help students construct representations that make sense to them, as was done in this phase (Onslow, 1991). Papert (1980) explains that anything can make sense to someone if they assimilate it into their mental models. This assimilation can be supported through the use of schematic representations.

Phase 5, schematic representations (2 weeks). During this phase students used paper and pencil to draw a schematic representation to solve the daily CGI-style word problem.

Justification for problem solving using schematic representations. Schematic representations act as a scaffold between concrete manipulations of problem elements and their numerical representations (Willis & Fuson, 1988). When students begin to visualize a problem through a schematic representation, students might need instruction on how to describe the actions and mathematical processes shown in the schematic representation at a mathematically symbolic level (Montague, n. d.). Problem model approach, which is similar to the visualization progression of this innovation, is an effective way of translating the mathematical problem into a mental image and then a schematic representation to come to an accurate solution (Hegarty et al., 1995). When students correctly create and label their schematic representations, they generally find the correct solution. Schematic representations seem to provide an organization of the problem elements and facilitate a correct mathematical process decision, which leads to an accurate solution (Willis & Fuson, 1988).

Phase 6, connecting schematic representations to writing number sentences (1 week). During this phase students used a hybrid solution strategy; they first solved the problem using a schematic representation on paper and then they wrote the number sentence that solved the problem. This hybrid phase was important because it is imperative to link mathematical symbolism to real world experiences whenever possible so students develop understanding of mathematics problems and their abstract symbols (Onslow, 1991). Care in instruction was taken during this phase because schematic drawings have been shown to be a successful solution strategy for second grade students when solving addition and subtraction word problems (Willis & Fuson, 1988), such as join and separate actions (Bebout, 1986), but students may struggle to write number sentences for problems whose semantic structure does not directly relate to the actions needed to solve the problem (DeCorte & Verschaffel, 1985).

Phase 7, writing number sentences (3 weeks). The final phase of the innovation lasted for three weeks. Students only used a number sentence to solve the daily word problem.

Justification for problem solving by writing number sentences. Much of traditional mathematics instruction and assessment focus on symbolic notation, usually in the form of a number sentence. Knowing what those numbers mean and represent in the real world requires mental constructions. Being able to solve word problems efficiently and with understanding are primary functions of mathematics education (Onslow, 1991). This is important to current mathematics instruction because the Common Core State

Standards guide teachers to creating classrooms where symbolic and abstract mathematical representations are commonplace (Common Core State Standards Initiative, 2010; White & Dauksas, 2012).

Word problem selection. Daily word problems used through this innovation period were selected from the 11 addition and subtraction problem types developed by Carpenter et al. (1999) and are displayed in Table 1. This study was not an instructionally maximal treatment because it did not focus solely on the problem types with which students were having the most problems (Willis & Fuson, 1988). All problem types were equally represented throughout the innovation with each problem type being practiced either five or six times by students. To control for problem ordering effects, the daily word problems were arranged so that no two same problem types were taught on back-to-back days and problem types were mixed throughout the innovation phases so students would not get accustomed to a problem and merely apply a practiced algorithm to solve it. Since CGI and NCTM posit that true problem solving is non-routine, this is an important aspect of this study (Carpenter et al., 1999; NCTM, 2000). Appendix C and Appendix D show all of the daily problem solving questions.

Problem solving procedure. This study took place over the course of 12 instructional weeks, encompassing 60 daily math problem solving lessons each approximately 30 minutes in length. Lessons began with a whole class reading of the day's problem. Students orally restated the problem and asked clarifying questions as needed. Word problems in Phases 1, 2, and 3 were read to the class; students did not receive a written copy and word problems in Phases 4, 5, 6, and 7 were read to the class and available in each student's Mathematics Problem Solving Journal (Appendix D).

Students were then briefly instructed on the solution strategy (acting out using realia, representing with manipulatives, creating a schematic representation, or writing a number sentence) they should use to solve the day's problem.

Partners worked together to solve each day's word problem. Student dyads spread out throughout the classroom, finding a working space. This portion of the daily problem solving procedure was termed "students-at-work" (Kline, 2008, p.145). Students were given up to 10 minutes to work on solving the problem using that phase's modeling strategy, though the time needed to solve each problem generally decreased over the innovation period. After dyads agreed on the solution they recorded their answer on the Daily Answer Recording Slip (Phases 1, 2, and 3) (Appendix E) or the Mathematics Problem Solving Journal (Phases 4, 5, 6, and 7). They used the remaining time in this 10 minute time period to rehearse how they would describe their solution strategy to the class if they were chosen to be that day's presenters.

When students were finished, they returned to their desks. Selected dyads were then called on to share their solution strategies with the class during a class "strategy conference" (Peter-Koop, 2005, p. 8). Strategy conferences allow students the opportunity to share their solution strategy, reflect on their work, and compare their strategy with others' strategies (Peter-Koop, 2005). Students were selected to present based on the manipulative they chose to use, how they modeled the problem, the schematic representation created, or the number sentence used to solve the problem. The students sharing their solution strategies served as MKOs and showed how they used their realia, manipulative, schematic presentation, or number sentence to solve the problem, explained how they knew to do these things to solve the problem, and answered questions their classmates posed. Classmates then asked clarifying questions, made agree/disagree statements, probed for further understanding by asking “what if” questions, and compared their solution strategy with the MKOs’ solution strategy making themselves actively engaged in their learning of mathematics (Reinhart, 2000). Seeing these new ways to solve problems allowed students to relate their solution strategy to classmates’ and find more efficient ways to solve problems (NCTM, 2000). Because the strategy conference is an imperative step leading to student understanding, visualizing different strategies, making generalizations, identifying inconsistencies in a person’s reasoning, and verifying a student’s own solution strategy it was not rushed (NCTM, 2004). The sharing and discourse process lasted between 10 and 20 minutes.

Justification for strategy conferences. This strategy conference process was a critical part of the innovation because it was designed to benefit all members of the classroom. It provided an opportunity to scaffold students with cognitively lower bottoms on their zone of proximal developments (ZPDs) (Vygotsky, 1978). Heuser (2005) found that when students with lower level solution strategies, such as pictures, observed peers with higher level solution strategies, such as number sentences, the students who had previously used a lower complexity solution strategy began showing understanding of the higher level solution strategies and began experimenting with them. This discussion time also benefitted the MKOs because when students reflect on their strategies and share them verbally, students deepen, develop, and extend their understanding of mathematical concepts (Burns & Silbey, 2001; Carpenter, Fennema, & Franke, 1996; Kline, 2008) and build an understanding of flexible, successful ways to solve problems (NCTM, 2000). By participating in strategy conferences, limited English proficient students get experience verbalizing their thought processes, becoming more comfortable and confident with their overall English language skills and mathematics discourse (Hoffert, 2009). Inter-peer conversations also give teachers a direct look at student thinking, reasoning, and logic (NCTM, 2004) and help the teacher design the next teaching steps (Drake et al., 2009; Kline, 2008).

Post-assessment. After all 12 weeks of the innovation period, each student was individually post-assessed using the pre-/post-assessment five question test. An identical testing format to the pre-assessment took place. Solutions were again recorded on the Solution Strategy Recording Form which was used in the analysis phase of the study. Administration of the post-assessment lasted between 7 and 14 minutes per student.

Data Collection Tools and Analysis

The four research instruments that were employed in this mixed-methods study were the Solution Strategy Recording Form (pre- and post-), the Daily Answer Recording Slips, the Mathematics Problem Solving Journal, and video recorded observations of daily problem solving dyads. Figure 6 shows the data collection matrix for this study.

Research Questions and Data Sources	Solution Strategy Recording Form (pre- and post-) Quan & Qual	Daily Answer Recording Slips Quan	Mathematics Problem Solving Journal Quan	Video recorded observations of dyads' problem solving Quan & Qual
1. How does a class of second grade students at San Marcos Elementary solve Cognitively Guided Instruction-style contextual word problems?	X			X
2. How and to what extent does partnered Cognitively Guided Instruction-style mathematics word problem solving through guided incremental steps affect a class of San Marcos second graders' mathematics problem solving abilities?	X	X	X	X

Figure 6. Relationship between the data collection instruments and research questions.

Care was taken when designing the data collection instruments and the innovation to maintain validity. First, to counter the influence of the practice effect, the innovation used all of the 11 different problem solving question types and did not strictly focus on the five assessed problem types or use parallel problems for the daily problem solving questions. Next, because pre-assessment took place during the first week of school, test anxiety and unfamiliarity with the test administrator (me) could have been additional threats to validity related to testing. To counter these threats, I used the first two days of the school year, before testing began, to allow the students to get comfortable with me, and I spent as much time as possible talking with the students. I also thoroughly explained the testing procedure to students in child-friendly terms before the pre-assessment began to minimize test anxiety caused by unfamiliarity with the testing procedure; a statement such as, "I will be asking you five math problem solving questions so I can get to know you better and learn how you solve math problems," was used. History could have been the greatest threat to validity in this study (Smith & Glass, 1987). This innovation was in addition to my usual mathematics instruction. I countered the history effect by not including additional similar style problem solving opportunities in the daily second grade level traditional math instruction students received; rather, daily traditional mathematics instruction focused other topics in mathematics related to the conceptual development of number sense and operation.

Solution Strategy Recording Form for the pre- and post-assessments. The Solution Strategy Recording Form for the pre-/post-assessment instrument was created to gather data for both Research Question 1 and Research Question 2, how a class of second grade students solve Cognitively Guided Instruction-style contextual word problems and how partnered Cognitively Guided Instruction-style mathematics word problem solving affects second graders' mathematics problem solving abilities. Quantitative and qualitative data were collected using the pre-/post-assessment and recorded on the Solution Strategy Recording Form. Quantitative data, in the form of correctness of student answer, solution strategy used, and solution strategy subset used, were collected. Correctness of solution was transferred to the Student Answer Correctness Chart (Appendix F) for ease of analysis. Then, all of the quantitative data were entered into Statistical Package for the Social Sciences (SPSS) computer-based data analysis software. Qualitative data, in the form of student's verbal solution strategy used or teacher's recorded notes on student's solution strategy, were written on the Solution Strategy Recording Form. As data were collected, data were transcribed using the Microsoft Word word processing program onto the Video Recording Observation Protocol (Appendix G).

Creation and administration of the pre-/post-assessment. The five problems in the pre-assessment were higher cognitive demand CGI-style addition or subtraction word problems, and they were problem types that students who enter my classroom generally have difficulties solving. CGI posits that there are 11 different addition and subtraction problem types (Carpenter et al., 1999). This pre-/post-assessment utilized five of these problem types: Join, Change Unknown (addition with the second addend missing); Join, Start Unknown (addition with the first addend missing); Separate, Change Unknown (subtraction with the subtrahend missing); Separate, Start Unknown (subtraction with the minuend missing); and Compare, Referent Unknown (addition or subtraction without an action in the problem's wording) (Carpenter et al., 1999). The five different problem types selected for the pre-/post-assessment have been found to be within second graders' zone of proximal development when they involve one- and two-digit numbers. Start Unknown, which was included in the pre-/post-assessment, and Compare Quantity Unknown, which was not included in the pre-/post-assessment, are the most difficult for students of this age to solve (Willis & Fusion, 1988). Additionally, the problem types in the assessment are similar to problems students would be expected to solve on the mathematics section of the Stanford 10 assessment. Figure 7 shows the solution strategy subtypes one would expect to be used by students taking this pre-/post-assessment.

Problem Type	Direct Modeling Strategies: Strategy Description	Counting Strategies: Strategy Description
Join (Change Unknown) Chuck had 3 peanuts. Clara gave him some more peanuts. Now Chuck has 8 peanuts. How many peanuts did Clara give to him?	*Joining To* A set of 3 objects is constructed. Objects are added to this set until there is a total of 8 objects. The answer is found by counting the number of objects added.	*Counting On To* A forward counting sequence starts from 3 and continues until 8 is reached. The answer is the number of counting words in the sequence.
Separate (Change Unknown) There were 8 people on the bus. Some people got off. Now there are 3 people on the bus. How many people got off the bus?	*Separating To* A set of 8 objects is counted out. Objects are removed from it until the number of objects remaining is equal to 3. The answer is the number of objects removed.	*Counting Down To* A backward counting sequence starts from 8 and continues until 3 is reached. The answer is the number of words in the counting sequence.
Join (Start Unknown) Deborah had some books. She went to the library and got 3 more books. Now she has 8 books altogether. How many books did she have to start with?	*Trial and Error* A set of objects is constructed. A set of 3 objects is added to the set, and the resulting set is counted. If the final count is 8, then the number of objects in the initial set is the answer. If it is not 8, a different initial set is tried.	*Trial and Error* A number is selected and a forward counting sequence starts from the number and continues until 8 is reached. If the count of numbers is 3, then the initial number is the answer. If the count of numbers is not 3, then a different initial number is tried.

Figure 7. Students' solution strategy subtype expected to be used and examples. Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 19 & 23), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright 1999 by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson.

The decision to include a reduced number of problems was due to time limitations of the study and an understanding of the developmental level of beginning second grade students; each question on the assessment took up to three minutes for the student to solve and asking students to solve 11 questions was not developmentally appropriate. I believed that adequate generalizations could be made based on student solution strategies and answers to the five questions. Additionally, five questions were approximately 45% of the problem types, which was large enough to be representative of the group of problems involved in the study. All five assessment questions were taken directly from *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 1999). Because this research study took place at the beginning of the school year, I had not provided students with in depth instruction on two-digit numbers over 20 nor any three-digit numbers before the pre-assessment was administered. Numbers in the assessment problems were kept below the number 20 so that understanding of the numbers would not affect student achievement on the pre-assessment nor post-assessment.

A researcher familiar with this study and qualitative and quantitative data collection tools, a curriculum specialist experienced in teaching second grade students, and a mathematics education professor reviewed the pre-/post-assessment before it was used with students. They checked for bias in the question wording, validity of the numbers contained in the problems, comprehensibility for a second grader, and effectiveness in obtaining data related to the research questions. Additionally, in January 2012, the pre-/post-assessment was piloted with a sample of five second grade students similar to the students who participated in this study. The pilot showed comprehensibility for second grade students and a developmentally appropriate amount of time needed to complete the assessment.

The assessment questions were selected with consideration to typical second grade student vocabulary and language understandings of FEP students who have recently been exited from the English language development program. During the assessment, I reread each question as many times as necessary. The student could choose any or none of the realia, manipulatives, or paper/pencil to help the student solve the problem. As the student was solving the problem, I recorded the student's actions taken to solve the problem, answer, correctness of the answer, solution strategy, and solution strategy subset on the Solution Strategy Recording Form. After the student told me the answer, if the student had not verbally or visibly solved the problem, I questioned using the prompt, "Please explain to me how you solved that problem." This was done so I could gain a full understanding of how the student solved the problem, because a student sometimes uses strategies that are not visible to the teacher, such as Counting On mentally or using Derived Facts mentally (Carpenter et al., 1999). Without asking the student to explain the solution strategy, valuable information could go unnoticed and unnoted. The Solution Strategy Recording Form was created to gain insight into how students solved problems before this innovation and how they solved problems after the innovation was implemented.

The Solution Strategy Recording Form was stapled into a packet for each student and contained a separate page for each assessment question. At the end of the study, each student had a Solution Strategy Recording Form packet for the pre-assessment and a separate recording form packet for the post-assessment. The packet was designed to facilitate collection of both qualitative and quantitative data. Qualitative data about how the child solved the problem was recorded as written field notes and student reflections on the lines behind the heading *student actions*. Quantitative data were collected from the headings *Is the student's answer correct?*, *solution strategies*, and *solution strategy subset*. The qualitative data recorded on this form helped investigate Research Questions 1 and 2. It shed light on how the second grade participants in this study solved CGI-style word problems and how participating in guided incremental problem solving steps affected students' problem solving abilities. The quantitative data recorded on this form helped answer Research Question 2, how and to what extent the innovation affects the correctness of students' problem solving solutions.

Daily Answer Recording Slips. The daily word problems posed to students during the first five weeks of the 12 week innovation period were answered on Daily Answer Recording Slips. During this time period, 25 different CGI-style word problems from the 11 CGI addition and subtraction problem types which were semi-randomly assigned to each phase of the innovation were asked. No two problems of the same type were asked back-to-back. Ten of these addition or subtraction word problems were asked during Phase 1 of the innovation, five during Phase 2, and 10 addition or subtraction word problems were asked during Phase 3. All problems were either directly stated in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 1999) or adaptations of these CGI problems, following the same format but including different values, names, and situations in the problems. To ensure reliability in the created problems, a teacher familiar with CGI reviewed all 25 contextual problems, checking their wording, numbers, and coherence within the CGI category that they were assigned.

The Daily Answer Recording Slips went hand-in-hand with the Mathematics Problem Solving Journals, and were used to collect data daily from each student which pertained to Research Question 2, how partnered CGI-style daily word problem solving through incremental steps affects students' problem solving abilities. Each student wrote a numerical answer on a Daily Answer Recording Slip. Quantitative data were collected from these slips in the form of correctness of students' answers. Data were entered into SPSS as either yes (correct) or no (incorrect) to be analyzed. The class mean was calculated, with an expected increase in the daily class mean to occur as the problem solving innovation progressed through the phases. From there, a paired-samples t-test was conducted to find the statistical significance of the change. Additionally, item analysis provided more insights into specific problem types. The Daily Answer Recording Slips were piloted in the Spring of 2012 with a second grade class similar to the study class. The pilot showed that the slips were comprehendible for second grade students and collected quantitative data reliably. Additionally, a researcher familiar with this study reviewed the slips and found no faults with their format.

Mathematics Problem Solving Journal. The Mathematics Problem Solving Journal was used during the final seven weeks of the 12 week innovation period, as a place for students to solve and record their solutions for their daily word problems. The benefits of having a Problem Solving Journal is that it provides students a place to record anything from simple drawings to get the problem solving process started all the way up to multiple solution strategies. By recording their solution strategy, students are able to recall their investigative work more easily when reporting their solution to the class (NCTM, 2004). The Mathematics Problem Solving Journal was set up in a book-like format. It was created by inserting pages with one question printed on each page into a folder using tangs. The directions for each page were printed at the top of the page followed by that day's contextual problem. For questions in Phase 4 and Phase 5 there was a large open portion in the middle of the page where students could draw their schematic representation of the problem. At the bottom of each page there was a line that started out with the word *Answer* and had a statement with a blank in it where students recorded their answer to that problem (Appendix D, p. 91 - 105). For the questions in Phase 6, there was also a line labeled with the words *Number sentence* where students wrote the number sentence they used to solve the problem (Appendix D, p. 106 - 110). For questions in Phase 7 of the innovation, there was no space for a schematic representation to be drawn, but there was space to record a number sentence as well as a space for the answer (Appendix D, p. 111 - 125).

The Mathematics Problem Solving Journal was analyzed to directly provide information about Research Question 2. It was used to collect quantitative data in the form of solution correctness of each problem. The Mathematics Problem Solving Journal was used to collect data for concurrent triangulation with data gathered through the Solution Strategy Recording Form for the pre- and post-assessments (Creswell, 2009) as well as a place for students to record their schematic representations of the daily word problems during Phase 4 and Phase 5 of the innovation and to record the number sentences students used to solve the daily word problems during Phase 6 and Phase 7.

The Mathematics Problem Solving Journal contained 35 different CGI-style word problems which had been semi-randomly assigned to each step of the innovation from the 11 CGI addition and subtraction problem types. No two problems of the same type were presented back-to-back in the Mathematics Problem Solving Journal. There were five addition or subtraction word problems for Phase 4, 10 addition or subtraction word problems for Phase 5, five addition or subtraction word problems for Phase 6, and 15 addition or subtraction word problems for Phase 7. All problems were either directly stated in *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 1999) or were adaptations of these CGI problems, following the same format but including different values, names, and situations in the problems. To ensure reliability in the created problems, a teacher familiar with CGI reviewed all 35 contextual problems, checking their wording, numbers, and coherence within the CGI category that they were assigned.

Video recorded observations. Video recordings were used to collect quantitative and qualitative data on Research Question 1, how a class of second grade students solves GGI-style word problems, and quantitative and qualitative data on Research Question 2, how and to what extent partnered Cognitively Guided Instruction-style mathematics word problem solving through guided incremental steps has an effect on class of San Marcos second graders' mathematics problem solving abilities. Once weekly, on Wednesdays, three dyads were video recorded during the students-at-work phase of the daily problem solving innovation. A Flip camera on a tripod was used to do the video recorded observations because this type of observation is less intrusive than traditional teacher observations (Creswell, 2009). Transcription of the dialogue and observation notes of the actions of the dyad onto the Video Recording Observation Protocol took place following the video recording. A benefit of video recorded observations is that they can provide completeness of analysis because videos and subsequent transcriptions can be observed numerous times through different foci (Erickson, 1986). I looked for how the students truly solved the problem. Did they solve the problem using that phase's solution strategy, or did they rely on another strategy? I also looked for how working with a partner impacted solutions and solution strategies. Video recordings also reduce observer on primitive analytic typification by making it easier to review material before making inferences about it (Erickson, 1986).

Observations are an effective data collection instrument, especially when looking at the actions of participants. Issues may arise with self reporting because of the maturity level of participants, cognizance of one's own actions, or subtleties of interactions between participants (Corbin & Strauss, 2008). During the mathematics problem solving study that I conducted in the Spring of 2011, which I mentioned previously, I found that there were discrepancies between what my second grade students reported they did and what they actually did to solve a problem.

Video recorded observation participants in this study were selected using rank order purposeful sampling and stratified random sampling but were not selected because of superior mathematical problem solving ability (Creswell, 2009; Gay, Mills, & Airasian, 2009). To select participants, dyads were ranked from highest to lowest in their problem solving abilities on the pre-assessment, based on the number correct and complexity of solution strategy used. The highest and the lowest dyads on the list were automatically selected as video recorded observation participants. The names of the members of the middle four dyads were then written on a slip of paper each and put into a hat. One group was randomly selected to participate in the video recorded observations as the medium ability group (Gay et al., 2009). This sample was three out of 11 dyads, which was about 27% of the population of this study. The goal of qualitative research sampling is to get to data saturation, and with this size sample, I think I did. In a pilot test of this data collection technique, conducted in February 2012, students were unfazed by the presence of the video camera and student-to-student interactions appeared authentic.

Role of the Researcher-Practitioner

Throughout this plan, I acted as a researcher-practitioner. My job was to serve as the translator of participants' words and actions. I observed, analyzed, interpreted, and reported from the participants to the reader (Corbin & Strauss, 2008). During different phases of innovation, I had different jobs. For example, when pre- and post-assessment data were being collected, I served as researcher and practitioner, interacting with students as a teacher while collecting study data. When video recorded observations were being conducted, I served as a complete observer, researcher, and practitioner, but not participant (Creswell, 2009). I did not engage with the dyad being recorded, as to not skew their selected solution strategy or final answer. When students were sharing their solution strategy with the class, I served as practitioner, facilitating the discussion as needed, but, on rare occasions, I also was a participant when no other MKOs were available to model that solution strategy (Stringer, 2007). This occurred three times throughout the innovation process.

Throughout the study, I served as the classroom teacher, as well as the designer, innovation implementer, assessor, and analyzer of the data. I prepared the pre-/post-assessment and CGI-style problem for each day, assessed each student using the pre-assessment, and paired the students for their daily problem solving tasks, as well as facilitated the in-class discussions through selecting the MKOs and kept the discussion flowing when needed. When the innovation had concluded, I reassessed all participants using the post-assessment, analyzed and categorized student solution strategies, recorded solution correctness on charts and in SPSS, coded, and analyzed the data and findings. I then compared and integrated data sources to create warranted assertions and reported them (Greene, 2007).

Mixed Methods Analysis

The quantitative and qualitative data from this study carried equal weight in the analysis process. Quantitative data were collected through four sources. First, the pre-assessment was administered with results recorded on the Solution Strategy Recording Form and transferred to the Student Answer Correctness Chart and the Student Answer Solution Strategy Chart (Appendix H). Second, the post-assessment was administered with results recorded on the Solution Strategy Recording Form and transferred to the Student Answer Correctness Chart and the Student Answer Solution Strategy Chart. Third, the students' daily problem solving answers were recorded on the Daily Answer Recording Slips and in the Mathematics Problem Solving Journal and then compiled on the Daily Problem Solving Answer Chart (Appendix I). Fourth, the video recorded weekly observations were analyzed for solution strategy used, problem type, number of words said, and length of problem solving, and these data were recorded on the Video Recorded Observation Dyads Transcription Data Chart.

Qualitative data were collected in three formats. First, three dyads solving mathematics word problems were video recorded weekly. These video recorded observations were transcribed onto the Video Recording Observation Protocol. Second, descriptions of the words and actions students used to solve problems on the pre-assessment were recorded on the Solution Strategy Recording Form and then transcribed using the Microsoft Word word processing program. Third, descriptions of students' words and actions used to solve post-assessment problems were recorded on the Solution Strategy Recording Form and transcribed using Microsoft Word in the same fashion as the pre-assessment qualitative data were collected and recorded. Figure 8 shows the data collection documents inventory.

Data	**Inventory**
Pre-assessment: Solution Strategy Recording Form	19 students x 5 pages of forms = 95 pages
Pre- and Post-assessment: Student Answer Correctness Chart	1 typed page
Pre- and Post-assessment: Student Answer Solution Strategy Chart	1 typed page
Pre-assessment: Solution Transcription Chart	8 typed single spaced pages
Post-assessment: Solution Strategy Recording Form	19 students x 5 pages of forms = 95 pages
Pre- and Post-assessment: Student Answer Solution Strategy and Strategy Subset Chart	1 typed page
Post-assessment: Solution Transcription Chart	8 typed single spaced pages
Daily Answer Recording Slips	19 students x 25 slips = 475 slips
Mathematics Problem Solving Journals	19 students x 35 pages = 665 pages
Daily Problem Solving Answer Chart	2 typed pages
Video Recorded Weekly Observations	1 hour 36 minutes
Video Recording Observation Protocol	3 groups x 13 observations x 1 typed single spaced page per observation = 39 typed single spaced pages
Video Recorded Observation Dyads Transcription Data Chart	3 typed pages

*Figure 8***.** Data collection inventory.

Quantitative. The impact of this mathematics innovation was gauged quantitatively by investigating the correctness of students' answers on the pre-assessment compared to the post-assessment, comparing the complexity of the solution strategies used during the pre-assessment and the post-assessment, looking at the percentage of problems students solved correctly on the first 20 daily problem solving questions as compared to the final 20 daily problems, and comparing the amount of time video recorded dyads spent solving problems and the number of words they said while solving problems at the beginning of the innovation to the end.

Pre-assessment and post-assessment. During pre- and post-assessments, students' answers were first recorded on the Solution Strategy Recording Form. From there, data were recorded on the Student Answer Correctness Chart using the Microsoft Word program as either a *Y* indicating that the answer was correct or an *N* indicating that the answer was incorrect. These correctness data were entered into SPSS and the means of the pre- and post-assessments were computed. A paired-samples t-test was conducted to find if the difference in performance was significant pre- to post-. Additionally, the number of problems each student solved correctly and incorrectly was computed and transformed into percents, the total percentage correct and incorrect per assessment question was calculated, and paired-samples t-tests were performed to find if the change in percentage correct from pre- to post- for each test question was statistically significant.

The Solution Strategy Recording Form that was used to record student answers on the pre- and post-assessments was also used to record the solution strategy and strategy subset that students used to solve each test question. CGI posits that students' solution strategies progress through the early elementary years from using Direct Modeling strategies, to Counting strategies, to Number Facts to solve problems. Progression through these phases vary from child to child, and children vary strategies based on the problem type they are presented (Carpenter et al., 1999). The solution strategy and solution strategy subset were circled on the table at the bottom of the Solution Strategy Recording Form. These data were then transferred onto the Student Answer Solution Strategy Chart in coded form. The first solution strategies were coded as *0* to indicate that no solution strategy was used or that a student guessed on the answer, *1* to indicate that a Direct Modeling strategy was used, *2* to indicate that a Counting strategy was used, or *3* to indicate a Number Facts strategy was used. Solution strategies developed in complexity with a 0 strategy being the least complex and a 3 strategy being the most complex. Strategy subsets were further coded using a second number, from 1 through 6. For example, a student who answered Assessment Question 1 using a Direct Modeling, Joining All strategy would be coded as 1-1. A student who answered the same question using a Number Facts, Recalled Fact strategy would have the answer coded as 3-2. The solution strategy subset number did not correspond with a higher level complexity in strategy subset used. Using this coding strategy allowed for ease of analysis using SPSS. Figure 9 displays the different solution strategies and solution strategy subsets that students may employ when solving CGI-style word problems, as well as the coding system used in this study.

Solution strategy	No specific solution strategy Code: 0	Direct Modeling Code: 1	Counting Code: 2	Number Facts Code: 3
Solution strategy subset	Guess or no answer Code: 0	Joining All Code: 1-1	Counting On From First Code: 2-1	Derived Fact Code: 3-1
		Joining To Code: 1-2	Counting On From Larger Code: 2-2	Recalled Fact Code: 3-2
		Separating From Code: 1-3	Counting On To Code: 2-3	
		Separating To Code: 1-4	Counting Down Code: 2-4	
		Matching Code: 1-5	Counting Down To Code: 2-5	
		Trial and Error Code: 1-6		

Figure 9. Solution strategies and strategy subsets with codes.

Differences in solution strategies and solution strategy subsets used on the pre-assessment and post-assessment were analyzed. First, the frequency of solution strategies was calculated for the entire pre-assessment and the entire post-assessment. These results were then checked for statistical significance through a paired-samples t-test using SPSS. Next, these data were further analyzed by calculating the frequencies of solution strategy used by assessment question. After, the frequencies of solution strategy subsets were calculated for the entire pre-assessment and the entire post-assessment. Then, these data were further analyzed by calculating the frequencies of solution strategy subsets used by students for each problem on the pre-assessment and post-assessment. Using these SPSS frequency tables, comparisons between pre-assessment and post-assessment solution strategy and solution strategy subsets were made.

Daily problem solving. Each day students were asked one CGI-style word problem, solved the problem using that phase's strategy, and recorded their answer on the Answer Recording Slip or in their Problem Solving Journal. Answers were then compiled on the Daily Problem Solving Answer Recording Chart in Microsoft Word. From there, the daily problem solving question answers were scored as either correct (1) or incorrect (0) and entered into SPSS. Analysis was conducted using these data to determine if there was a statistical difference between the average correctness of students' answers from the first third of the innovation (20 problems) and the final third of the innovation (20 problems) using a paired-samples t-test.

Video recorded observations. Three dyads, a high ability group, an average ability group, and a low ability group, were video recorded solving their daily problem solving word problem one time each week. These video recorded observations were viewed and data from them were entered on the Video Recorded Observation Dyads Transcription Data Charts (Appendix J) using Microsoft Word. This chart contained information on the problem type, the correctness of the answer, the number of words the dyad said while solving the problem, and the length of time it took the dyad to solve the problem for each observed word problem. Length of time to answer questions in seconds was entered into SPSS, as well as the number of words the dyads said while solving each problem. The mean length of time it took each of the three dyads to solve problems in Phase 1 was calculated and compared with the mean length of time it took each dyad to solve problems in Phase 7. A paired-samples t-test was conducted to find significance of the difference between the amount of time it took dyads to complete problem solving questions at the beginning of the innovation and the end of the innovation. Then the mean number of words spoken during Phase 1 for each dyad was calculated and the mean number of words spoken in Phase 7 for each dyad was calculated. The difference in number of words spoken during daily problem solving from the beginning of the innovation to the end of the innovation was analyzed using a paired-samples t-test.

Qualitative. Qualitative data were collected during this study to describe how students solved CGI-style word problems, to help explain why students solved problems in the ways they did, to find the effect of the innovation on students' problem solving strategies, and to shed light on the interactions between members of the problem solving dyads.

Pre-assessment and post-assessment. When administering the pre-assessment and the post-assessment, I paid close attention to how students solved the problems and wrote down everything they said and did while deriving their solution on the Solution Strategy Recording Form. This information was then transcribed in a Microsoft Word chart called the Pre- and Post-assessment Solution Transcription Chart (Appendix K). Constant comparative method was employed to analyze students' strategies used to solve problems on the pre-assessment and the post-assessment. Constant comparative method of data analysis allows two sets or sources of data to be compared and similarities and differences to be found (Corbin & Strauss, 2008). First, students' verbal and nonverbal strategies were read individually and memos on sticky notes were written noting commonalities among the students' solution strategies. Transcriptions of strategies were reread and the important phrases were marked. During this process, I asked myself questions about the data to find out what the participant's responses really meant–the true meaning of their words beyond a surface level–to make sure I was interpreting them correctly and painting an accurate picture of what the student intended to say and what the data as a whole said (Corbin & Strauss, 2008). The reading and rereading process continued for all of the participants' data, making memos on sticky notes of commonalities. Open coding–categorizing the list of phrases and actions, forming basic groups that were related, such as strategies, emotions, actions, and levels of understanding–of the important words and phrases occurred (Corbin & Strauss, 2008; Glaser & Strauss, 1967). The categories were operationalized by defining what each category specifically meant and encompassed. To ensure reliability, another evaluator was enlisted to spot-check the phrase lists, looking for accuracy in, understanding of, and agreement with the categories. When agreement was met, preliminary codes–labels given to organized groups of data–for the categories were created and two or three letter abbreviations were assigned for each code (Gay et al., 2009). Again, to ensure reliability, another evaluator was enlisted to create codes for the lists of phrases, and our lists of codes were compared. The most appropriate codes for this data analysis were agreed upon. The data were coded by writing the code two-letter abbreviations above the key phrases circled in the observation data. Lists of all of the phrases for each code were made and the lists were enumerated by counting number of times each code appeared in the data to determine stability (Johnson & Christensen, 2004). Based on this open coding, axial coding–relating preliminary codes to each other creating larger, more robust codes–took place. During axial coding, all preliminary codes were merged into codes that encompassed all students' solution strategy words and actions and could be related to theoretical models (Corbin & Strauss, 2008; Glaser & Strauss, 1867). All of the coded data and categories were imported into a Microsoft Word chart titled Categories Pre- and Post-assessment Solution Strategies (Appendix L) under the headings *code*, *category*, *definition*, and *examples*. The chart was read and altered repeatedly until saturation occurred. From there, selective coding was used to weave a relationship between categories; the axial codes were used to create the core codes for pre- and post-assessment student problem solving (Corbin & Strauss, 2007). In this final step, data were analyzed and reduced to descriptive form, creating themes, which were recorded on sticky notes (Greene, 2007). From the sticky notes, a finalized bulleted list of themes was created. An example of a theme is, "Checking over work."

Video recorded observations. Once a week three dyads participated in video recorded observations by having their students-at-work portion of the day's problem solving process recorded. Their video recorded observations were watched and transcribed into the chart titled Video Recording Observation Protocol using the Microsoft Word program. These transcriptions included both what the students said and what they did while solving their daily word problems, as well as my reflective notes on their problem solving process. The video recorded observations or transcription of the observations are not data themselves; what is done with them is what constitutes data (Erickson, 1986), so constant comparative method was used to analyze the qualitative video recorded observations field notes (Corbin & Strauss, 2008; Strauss & Corbin, 1998). First, the actions and words of the dyads' solution strategies were read and reread. From doing this, it was found that the beginning of the innovation observations contained strategies that were too different among dyads to effectively combine, so only the transcription from the last phase of the innovation, Phase 7, was included in the coding process at this time. The process for creating categories, axial codes, and themes developed in an almost identical process to the way in which the pre- and post-assessment qualitative data were analyzed. Important phrases were circled and a list was made of those phrases. Open coding was done by making a list of the circled important phrases and then categorizing them into related groups, such as reasons, emotions, or actions.

The categories were operationalized so that a clear idea of each category was created. Another evaluator was again enlisted to spot-check the phrase lists, looking for accuracy in, understanding of, and agreement among the categories. Codes for the categories were created and two or three letter abbreviations were assigned to the codes. The other evaluator checked the codes for inclusiveness and accuracy. Lists containing all of the phrases for each code were created and the number of times each code appeared in the data was counted to determine stability. During this step, two similar codes with low usage counts were combined. The open codes and examples were entered into a Microsoft Word chart titled Categorized Video Recorded Observation Data Form (Appendix M). Axial codes were created by merging open codes and examples and creating codes that were suitably inclusive for all dyads' daily problem solving solution actions and words. Then, on sticky notes, memos were written about the relationships between codes. All of the sticky notes with memos and statements about dyads' problem solving strategies were gathered and reread. The information on the sticky notes was interpreted and bullet points of traits generated from the information were created. This bulleted list of traits was then used to create a vignette describing the typical problem solving approach students in this study might take during Phase 7 of this innovation.

The same method was then used to create traits for the three dyads separately for the beginning of the innovation video recorded observation field notes. As stated previously, the strategies, words, and actions employed by the dyads at the beginning of the innovation could not be analyzed together because differences among dyads were so great that combining field notes would have negated important individual problem solving traits. Understanding of individual dyad's true problem solving strategies and skills would have been lost. Therefore, the Phase 1 and Phase 2 video recorded observation transcriptions were analyzed separately by dyad. The constant comparative method was used for analysis of all three dyads' field notes in a nearly identical fashion to the way the Phase 7 data were analyzed. Through this analysis process, five or six traits were created for each dyad. These traits were used to create vignettes that depict how each dyad might solve a CGI-style word problem at the beginning of the innovation.

Timetable of the Study

Figure 10 shows a timetable of the study, including implementation of the innovation steps, data collection methods and times, data analysis methods and times, and data reporting procedures and times.

Sequence	Actions	Data
July 25, 2012- July 27, 2012	Pre-assessed	Data from student responses to and solution strategies used on the pre-assessment recorded on Solution Strategy Recording Form (quantitative and qualitative)
July 27, 2012- August 17, 2012	Recorded correctness of student answers Recorded student solution strategies	Pre-assessment answers and scores recorded on the Student Answer Correctness Chart (quantitative) Pre-assessment solution strategy and strategy subtypes recorded and coded on the Student Answer Solution Strategy Chart (quantitative) Quantitative data input into SPSS Words and actions of students' pre-assessment solution strategies transcribed into Solution Transcription Chart (qualitative)
July 28, 2012- July 29, 2012	Conducted preliminary data analysis to form daily problem solving dyads	Counted each student's total number correct and solution strategy and strategy subset used on pre-assessment (quantitative)
July 30, 2012- August 10, 2012	Implemented Innovation Phase 1 Video recorded observations of dyads	Students' answers written on Daily Answer Recording Slips (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative)

(figure continues)

Sequence	Actions	Data
August 13, 2012- August 17, 2012	Implemented Innovation Phase 2 Video recorded observations of dyads	Students' answers written on Daily Answer Recording Slips (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative)
August 20, 2012- August 31, 2012	Implemented Innovation Phase 3 Video recorded observations of dyads	Students' answers written on Daily Answer Recording Slips (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative)
September 3, 2012- September 7, 2012	Implemented Innovation Phase 4 Video recorded observations of dyads	Students' answers written in Mathematics Problem Solving Journals (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative)
September 10, 2012- September 21, 2012	Implemented Innovation Phase 5 Video recorded observations of dyads	Students' answers written in Mathematics Problem Solving Journals (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative)
September 24, 2012- September 28, 2012	Implemented Innovation Phase 6 Video recorded observations of dyads	Students' answers written in Mathematics Problem Solving Journals (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative)

(figure continues)

Sequence	Actions	Data
October 1, 2012- November 7, 2012	Implemented Innovation Phase 7 Video recorded observations of dyads	Students' answers written in Mathematics Problem Solving Journals (quantitative) Transcribed observations on the Video Recording Observation Protocol (qualitative) Length of problem solving and number of words recorded on Video Recorded Observation Dyads Transcription Data Chart (quantitative and qualitative)
November 8, 2012- November 13, 2012	Post-assessed	Data from student responses to and solution strategies used on the post-assessment recorded on Solution Strategy Recording Form (quantitative and qualitative)
November 14, 2012- November 30, 2012	Recorded correctness of student answers on post-assessment Recorded student solution strategies on post-assessment	Post-assessment answers and scores recorded on the Student Answer Correctness Chart (quantitative) Post-assessment solution strategy and strategy subtypes recorded and coded on the Student Answer Solution Strategy Chart (quantitative) Quantitative data input into SPSS Words and actions of students' post-assessment solution strategies transcribed into Solution Transcription Chart (qualitative)

(figure continues)

Sequence	Actions	Data
December 6, 2012- January 7, 2012	Completed data analysis	Compiled correctness of students' daily problem solving answers on Daily Problem Solving Answer Chart (quantitative) Ran descriptive and t-tests in SPSS (quantitative) Coded data and created themes using constant comparative method and recorded on Categorized Video Recorded Observation Data form and Categories Pre- and Post-Assessment Solution Strategies (qualitative)
January 8, 2012- January 17, 2013	Created and warranted assertions	Used Erickson's modified method of analytic induction to create and warrant assertions (quantitative and qualitative)
January 18, 2013- March 17, 2013	Prepared written findings	Compiled all findings into written report (quantitative and qualitative combined)
March 29, 2013	Defended dissertation	Formally presented all findings and assertions about study (quantitative and qualitative)

Figure 10. Study timetable.

CHAPTER 4

RESULTS

The purpose of this study was to investigate the effects of a guided mathematics problem solving innovation, focused on progressing solution strategy complexity through incremental steps, on the problem solving skills of a class of second grade students, as well as to investigate how these students solved mathematics word problems. This chapter discusses the analysis results of the data collected to provide findings for these research questions.

Analysis Process

A mixed methods research design, as was employed by this study, provides the potential for the better understanding of a phenomena and more detailed results for a research problem (Creswell, 2009). Quantitative and qualitative data were collected concurrently and then analyzed separately, as described in Chapter 3. Quantitative analysis of the pre- and post-assessment correctness, pre- and post-assessment solution strategy complexity, comparison of the correctness of the first 20 daily word problems compared to the final 20 word problems, the length of the students-at-work portion of the weekly video recorded dyads at the beginning of the innovation and the end of the innovation, and the number of words spoken by video recorded dyads during the students-at-work phase comparing the beginning of the innovation and the end of the innovation were done using paired-samples t-tests. Qualitative analysis employed by this study was grounded theory. Words and actions from the pre-assessment and post-assessment and video recorded dyads' weekly observations were coded using open, axial, and selective coding (Corbin & Strauss, 2008; Strauss & Corbin, 1998). Words and actions from the pre-assessment and post-assessment as well as video recorded dyads were compared using constant comparative method. This allowed for similarities and differences between the data sets on the same topic to be found. After, assertions for the pre- and post-assessment data were created and traits for video recorded dyads were formed.

Quantitative data results. Quantitative data in this study took the form of a pre- and post-assessment, solution strategy and strategy subset, comparison of the correctness of the first 20 daily problem solving questions to the last 20 daily problem solving questions, the length of the students-at-work portion of the weekly video recorded problem solving sessions, and the number of words students said while working in dyads to solve the daily word problems. Data were entered into Microsoft Word charts first for preliminary analysis and then further analyzed using SPSS, following the formats previously stated. This quantitative data will be combined with qualitative data from the study to make assertions about the effects of this study and to shed light on the study's research questions.

Pre- and post-assessment. As described previously, a problem solving assessment was administered before implementation and again after the implementation concluded. Students' answers, solution strategies and strategy subsets, and actions and words used when solving the problem were recorded.

Answer correctness. Students scored an average of 33.68% correct on the pre-assessment and 96.84% on the post-assessment. This was an increase of 63.16%. The type of question did not have an impact on student performance with significant increases in performance occurring for all questions. A paired-samples t-test was conducted to determine if the difference between pre-assessment scores and post-assessment scores was statistically significant. The result indicated that the mean post-assessment score (M = 0.9684, SD = 0.07) was significantly greater than the mean pre-assessment score (M = 0.3368, SD = 0.29), $t(18) = 9.66$, $p < .001$. Table 1 displays the percentage correct per assessment question, change in percentage correct from the pre-assessment to the post-assessment, as well as pre-assessment to post-assessment paired-samples t-test results per item on the assessment and the entire assessment.

Table 1

Paired-samples t-test Comparison in Means from Pre-assessment to Post-assessment by Assessment Question

Question	Mean			SD	95% CI	t(18)	p
	Pre-	Post-	Difference		LL UL		
1	42.10%	100.00%	+57.90%	0.51	[0.82, 0.33]	4.98	< .001
2	36.84%	100.00%	+63.16%	0.50	[0.87, 0.39]	5.56	< .001
3	47.37%	100.00%	+52.63%	0.51	[0.77, 0.28]	4.47	< .001
4	15.79%	94.74%	+78.95%	0.42	[0.99, 0.52]	8.21	< .001
5	26.32%	89.47%	+63.15%	0.60	[0.92, 0.34]	4.61	< .001
Total assessment	33.68%	96.84%	+63.16%	0.28	[0.77, 0.49]	9.66	< .001

Note. CI = confidence interval; LL = lower limit; UL = upper limit.

A comparison between student pre-assessment data and post-assessment data is shown in Table 2. As seen from this table, all students increased their percentage correct from the pre-assessment to the post-assessment.

Table 2

Pre- and Post-assessment Student Answer Correctness Chart by Student

Student	Question Number										% Correct		%
	1		2		3		4		5				
ID #	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-	Change
1	N	Y	N	Y	Y	Y	N	Y	Y	Y	40%	100%	+60%
2	Y	Y	N	Y	N	Y	N	Y	Y	Y	40%	100%	+60%
3	N	Y	N	Y	N	Y	N	Y	Y	N	20%	80%	+60%
4	Y	Y	N	Y	Y	Y	N	Y	N	Y	40%	100%	+60%
5	Y	Y	N	Y	Y	Y	N	N	N	Y	40%	80%	+40%
6	Y	Y	Y	Y	N	Y	N	Y	N	Y	40%	100%	+60%
7	Y	Y	N	Y	N	Y	N	Y	N	Y	20%	100%	+80%
8	N	Y	N	Y	N	Y	N	Y	N	N	0%	80%	+80%
9	N	Y	N	Y	N	Y	N	Y	N	Y	0%	100%	+100%
10	N	Y	N	Y	N	Y	N	Y	N	Y	0%	100%	+100%
11	N	Y	N	Y	N	Y	N	Y	N	Y	0%	100%	+100%
12	N	Y	N	Y	N	Y	N	Y	N	Y	0%	100%	+100%
13	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	80%	100%	+20%
14	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	80%	100%	+20%
15	Y	Y	N	Y	Y	Y	N	Y	N	Y	60%	100%	+40%
16	N	Y	Y	Y	Y	Y	N	Y	N	Y	40%	100%	+60%
17	Y	Y	Y	Y	Y	Y	N	Y	N	Y	60%	100%	+40%
18	N	Y	N	Y	N	Y	N	Y	N	Y	0%	100%	+100%
19	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	80%	100%	+20%

Note. Y = Correct; N = Incorrect.

Solution strategy complexity. Pre- and post-assessment results were also analyzed to find the primary solution strategy and strategy subset used to solve each assessment question. Overall, a paired-samples t-test indicated a statistical difference between the strategies students used to solve problems on the pre-assessment and post-assessment. More complex strategies were used on the post-assessment (M = 1.21, SD = 0.58) than the pre-assessment (M = 2.08, SD = 0.13), $t(95) = 8.08$, $p < .001$. This held true for each of the problem types individually, as well. The more complex strategies were significant at the $p < .05$ level on the post-assessment for all problem types. Table 3 shows the results of the paired-samples t-test.

Table 3

Paired-samples t-test Comparison for Student Answer Complexity Means from Pre-assessment to Post-assessment

Question	M (SD)	95% CI LL UL	t(18)	p
1	1.00 (0.94)	[1.45, 0.55]	4.62	< .001
2	1.21 (0.98)	[1.68, 0.74]	5.40	< .001
3	0.58 (1.17)	[1.14, 0.02]	2.16	< .045
4	1.05 (1.08)	[1.57, 0.53]	4.25	< .001
5	0.53 (1.02)	[1.02, 0.03]	2.25	< .037

Note. CI = confidence interval; LL = lower limit; UL = upper limit. 0 = Guess or no solution strategy; 1 = Direct Modeling strategy; 2 = Counting strategy; 3 = Number Facts strategy. For reference, the Question 1 mean of 1.00 indicates that students increased the complexity of their strategy by one level from the pre-assessment to the post-assessment.

Table 4 displays a summary of the primary solution strategies students used on the pre- and post-assessments and shows the change in their prevalence. The entire Student Answer Solution Chart can be seen in Appendix N.

Table 4

Summary of Students' Solution Strategies Used on Pre-assessment and Post-assessment

Solution Strategy	Pre-assessment	Post-assessment	Change in Prevalence
No Specific Strategy	8.42%	0.00%	-8.42%
Direct Modeling	73.68%	37.89%	-35.79%
Counting	6.32%	15.79%	+9.47%
Number Fact	11.58%	46.32%	+34.74%

As shown in the table, 8.42% of questions on the pre-assessment were answered using no specific strategy, with a student immediately guessing the answer or not stating a numerical answer, whereas no post-assessment questions were immediately answered using no specific strategy. An example of this was when Student 3 answered pre-assessment Question 2 by saying, "The answer is 3 because I just guessed," or Student 11 answering pre-assessment Question 2 by stating that the answer was, "Some books. I thought of it in my head." This is in contrast to the post-assessment where all students attempted to solve all problems and no students gave a non-numerical answer or immediately guessed at an answer without first trying to solve the problem. Additionally, analysis showed that on the pre-assessment Direct Modeling was by far the most common solution strategy employed by students, with 73.68% of the solutions being derived by a Direct Modeling strategy, whereas on the post-assessment, 37.89% of the solutions were derived from a Direct Modeling strategy. Number Facts was the most common solution strategy used on the post-assessment, with 46.32% of students solving problems using a Number Facts strategy.

Table 5 shows the percentage that each solution strategy was used by students to solve the pre- and post-assessment questions. The Direct Modeling strategy (Level 1) was used most commonly on all five pre-assessment questions, but was only the most common strategy used on two post-assessment questions. These were Question 3, which was a Separate, Change Unknown problem, and Question 5, which was a Compare, Referent Unknown problem. Number Facts (Level 3) was the most common strategy used on three of the five post-assessment questions. These were Join, Change Unknown, Join, Start Unknown, and Separate, Start Unknown problem types. In addition, no students used a Number Facts strategy on Question 1 on the pre-assessment and 42.20% (n = 8 out of 19) of the students used a Number Facts strategy to solve Question 1 on the post-assessment. Further, all pre-assessment questions had at least one student derive the answer through an immediate guess, but on the post-assessment, no students derived an answer through an immediate guess on any assessment problem. Complete solution strategies and the specific strategy subsets used by each student in this study can be seen in Appendix M.

Table 5

Solution Strategy Used to Solve Each Pre- and Post-assessment Question Shown in Percentages

	Pre- 1	Post- 1	Pre- 2	Post- 2	Pre- 3	Post- 3	Pre- 4	Post- 4	Pre- 5	Post- 5
Level 0	5.3	0.0	10.5	0.0	10.5	0.0	10.5	0.0	5.3	0.0
Level 1	84.2	36.8	68.4	15.8	63.2	47.4	68.4	31.6	84.2	57.9
Level 2	10.5	21.1	0.0	15.8	10.6	15.9	10.6	10.6	0.0	15.9
Level 3	0.0	42.2	21.1	68.5	15.8	36.8	10.6	57.9	10.5	26.3

Note. 0 = Guess or no solution strategy; 1 = Direct Modeling strategy; 2 = Counting strategy; 3 = Number Facts strategy. Columns may not add to 100.00 due to rounding.

Table 6 looks at the students' strategies more exhaustively by examining the strategy subset used on the pre-assessment and post-assessment. As shown in this table, all subsets of Strategies 0 (no solution strategy) and 1 (Direct Modeling), considered to be the more basic strategies, decreased in prevalence between the pre- and post-assessments, whereas all of the subsets of Strategy 3 (Number Facts), considered to be the most advanced strategy, increased in frequency. The two greatest changes in percentage of solution strategy subsets used from the pre-assessment to the post-assessment occurred in the Level 1 (Direct Modeling) and the Level 3 (Number Facts) strategies. The percentage of assessment problems solved using the Number Facts, Recalled Fact strategy subset increase 27.37% and the percentage of assessment problems solved using the Direct Modeling, Joining All strategy subset decreased 13.68%. Students' individual solution strategy subset used on each assessment question can be seen in Appendix O.

Table 6

Solution Strategy Subsets Used During Pre-assessment and Post-assessment

Solution Strategy Subset	% of Pre-assessment Solutions	% of Post-assessment Solutions	Change in Prevalence
0: No Solution Strategy	8.42%	0.00%	-8.42%
1.1: Direct Modeling, Joining All	13.68%	0.00%	-13.68%
1.2: Direct Modeling, Joining To	14.74%	8.42%	-6.32%
1.3: Direct Modeling, Separating From	21.05%	10.53%	-10.52%
1.4: Direct Modeling, Separating To	6.32%	7.37%	+1.05%
1.5: Direct Modeling, Matching	5.26%	5.26%	0.00%
1.6: Direct Modeling, Trial and Error	12.63%	6.32%	-6.31%
2.1: Counting, Counting On From First	2.11%	1.05%	-1.06%
2.2: Counting, Counting On From Larger	0.00%	2.11%	+2.11%
2.3: Counting, Counting On To	2.11%	8.42%	+6.31%
2.4: Counting, Counting Down	1.05%	2.11%	+1.06%
2.5: Counting, Counting Down To	1.05%	2.11%	+1.06%
3.1: Number Facts, Derived Fact	3.16%	10.53%	+7.37%
3.2: Number Facts, Recalled Fact	8.42%	35.79%	+27.37%

Note. Columns may not add to 100.00% due to rounding.

When looking at the solution strategy subsets used to solve each assessment question, only Assessment Question 1 and Assessment Question 2 had at least one of the same strategy subsets used most commonly on the pre-assessment also used most commonly on the post-assessment. When solving all other questions, students relied on different strategies on the post-assessment than they did on the pre-assessment. Number Facts, Recalled Fact was one of the most common solution strategy subsets used to solve four out of the five questions on the post-assessment. Table 7 shows this information by presenting the strategy and subset used most commonly broken out by question.

Table 7

Strategy and Strategy Subset Most Frequently Used on the Pre-assessment and Post-assessment by Question

Question	Pre-assessment Strategy & Subset	Post-assessment Strategy & Subset
1	Direct Modeling, Joining To	Direct Modeling, Joining To
2	Direct Modeling, Joining All; Number Facts, Recalled Fact	Number Facts, Recalled Fact
3	Direct Modeling, Separating From	Direct Modeling, Separating To; Number Facts, Recalled Fact
4	Direct Modeling, Separating From	Number Facts, Recalled Fact
5	Direct Modeling, Trial and Error	Direct Modeling, Separating From; Direct Modeling, Matching; Number Facts, Recalled Fact

Note. Assessment questions with more than one strategy and strategy subset listed indicates that an equal number of assessment questions were solved using those solution strategies and strategy subsets.

Table 8 displays the different types of CGI-style addition and subtraction word problems employed by this pre-/post-assessment. It further shows the solution strategies and the strategy subsets that CGI posits students are most likely to use when solving these types of problems and the strategies and strategy subsets students in this study most often used.

Table 8

Problem Types and Solution Strategies and Strategy Subsets Most Commonly Used by Primary-Aged Children and Participants in this Study on the Post-assessment

Problem	Direct Modeling		Counting		Number Facts	
	CGI	Actual	CGI	Actual	CGI	Actual
1: Join, Change Unknown	Joining To	Joining To	Counting On To		Derived Facts	
2: Join, Start Unknown	Trial and Error	Joining All	Trial and Error			Recalled Facts
3: Separate, Change Unknown	Separating To	Separating From	Counting Down To			
4: Separate, Start Unknown	Trial and Error	Separating From	Trial and Error			
5: Compare, Referent Unknown	**	Trial and Error	**			

Note. ** indicates that there is not a primary strategy used to solve that type of problem. Children generally use Joining To, Separating From, Counting On To, or Counting Down To strategy subsets to solve these problems. Empty cells indicate that the most common strategy subset used to solve this type of problem was not in that solution strategy. Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 25), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright 1999 by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson.

As seen from this table, Assessment Question 1 was the only problem in which students participating in this study used the same solution strategy that CGI posited students would use. Additionally, when CGI posited that most students would use the Direct Modeling, Trial and Error strategy and subset to answer a problem type, Trial and Error was not the most common solution strategy and subset used by students in this study.

Daily problem solving answers. As another way of determining the impact of this innovation, the students' correctness on the first 20 daily problem solving questions (first third of implementation) was compared to the correctness on the last 20 daily problem solving questions (last third of implementation). The mean percent correct on the first 20 daily problem solving questions students solved was 75.01%, and the mean of the last 20 daily problem solving questions students solved was 85.34%. This was an increase of 10.33% between the first 20 problems and the last 20 problems students solved. Additionally, a paired-samples t-test was conducted to find the significance of the increase in the percentage of daily problem solving questions solved correctly. The results indicated that the average correctness of the set of the last 20 daily problem solving questions ($M = 0.86, SD = 0.11$) was significantly greater than the average correctness of the set of the first 20 daily problem solving questions ($M = 0.75, SD = 0.12$), $t(18) = 4.52$, $p < .001$. The 95% confidence interval for the average difference between the two problem sets was 0.15 and 0.06. This t-test shows that the increase in student daily problem solving performance likely did not occur by chance and instead can be associated with the innovation. Table 9 shows the comparison of correctness between the two thirds of the implementation period by student.

Table 9

Comparison of the Percent Correct of the First 20 Daily Problem Solving Questions to the Last 20 Daily Problem Solving Questions

Student ID	First 20 Problems	Last 20 Problems	Change in % Correct
1	89.47%	80.00%	-9.47%
2	80.00%	95.00%	+15.00%
3	57.89%	84.21%	+26.32%
4	80.00%	90.00%	+10.00%
5	78.95%	78.95%	0.00%
6	75.00%	100.00%	+25.00%
7	70.00%	85.00%	+15.00%
8	55.00%	58.82%	+3.82%
9	64.71%	85.00%	+20.29%
10	65.00%	80.00%	+15.00%
11	58.82%	76.47%	+17.65%
12	68.42%	70.00%	+1.58%
13	95.00%	95.00%	0.00%
14	89.47%	100.00%	+10.53%
15	88.89%	88.89%	0.00%
16	78.57%	84.21%	+5.64%
17	75.00%	100.00%	+25.00%
18	65.00%	75.00%	+10.00%
19	90.00%	95.00%	+5.00%

Note. Students who were absent from the classroom and did not answer a question did not have that day's solution marked correct or incorrect.

Video recorded observation problem solving lengths. Over the course of the innovation, the length of time the video recorded dyads spent solving their daily word problem was recorded. Comparisons between the beginning of the innovation problem solving times and the end of the innovation problem solving times were made. Analysis showed that the average length of time it took the video recorded dyads to solve the Phase 1 problems was 2 minutes and 54 seconds. The average length of time it took for the dyads to solve the Phase 7 problems was 2 minute and 16 seconds. A paired-samples t-test was conducted to determine if there was a statistical difference in problem solving lengths between the beginning of the innovation and the end of the innovation. For this test results indicated that the problem solving length of the first phase ($M = 174$ seconds, $SD = 27.71$) was not significantly longer than the problem solving length of the last phase ($M = 136$ seconds, $SD = 22.27$), $t(2) = 2.01$, $p > .10$. The 95% confidence interval for the average difference between the two phases was 43.33 and 119.33. Though there was not a statistical significance between the two phases of the innovation, the problem solving lengths for all three dyads decreased over the innovation period, with an average decrease of 21.84% when comparing Phase 1 to Phase 7. The results not being significant may have been due to small sample size since n = 3.

Video recorded observation number of words spoken during problem solving. During weekly video recorded observations of each dyad, the number of words spoken during the students-at-work portion of the daily problem solving routine was recorded so comparisons between the beginning and end of the innovation could be made. The mean number of words students spoke while solving a daily problem was calculated and a paired-samples t-test was conducted to find if the number of words spoken during Phase

1 was statistically different than the number of words spoken during Phase 7. Paired-samples t-test results showed that there was not a statistical difference between the number of words said during problem solving in Phase 1 (M = 111.67, SD = 22.41) and Phase 7 (M = 80.67, SD = 17.53), $t(2) = 1.64$, $p > .20$. The 95% confidence interval for the average difference between the two phases was 50.30 and 112.30. The overall change in number of words said during the daily students-at-work portion of the lesson, however, was a decrease of 27.76%. The lowest ability dyad showed a decrease of 33.33% and the highest ability dyad showed a decrease of 45.45%. These results seemed more on trend with the overall class's performance, whereas the medium ability dyad increased their number of words said during the students-at-work portion by 2.90%. This dyad was less confident about their problem solving abilities at the beginning of the innovation and their confidence appeared to grow throughout the innovation.

Qualitative data results. The actions and words from the pre-assessment and post-assessment and the video recorded weekly problem solving observations were analyzed using constant comparative method, as described previously. Through this process themes, subthemes, and assertions were created (Strauss & Corbin, 1998), and enumeration allowed temporal words to be added to assertions with accuracy because codes were counted to check for stability and usage (Johnson & Christensen, 2004). This helped me state the degree to which certain events or responses occurred when stating my assertions.

Pre- and post-assessment words and actions. Analysis of the words and actions students used when solving pre- and post-assessment questions showed various similarities and differences between their problem solving strategies and skills before the innovation implementation and after the innovation. Using grounded theory, themes were created to show these comparisons (Corbin & Strauss, 2008). Table 10 shows the themes, theme related-components, and assertions that can be made relating to similarities between the pre- and post-assessment qualitative data.

Table 10

Pre- and Post-assessment Similarities Themes, Theme Related Components, and Assertions

Themes	**Theme Related Components**	**Assertions**
Breaking problem into parts can create success	Separating problems into steps helped lower-ability students. Students asked researcher to stop reading as needed and did the problem step by step.	When problems were presented orally, breaking them down into digestible parts made them more accessible to all students.
Describing the solution process	Students talked during problem solving equally on the pre-assessment and post-assessment. Students willingly explained steps used to get their answer after solving. Students explained how they solved the problem more when they did not solve the problem in their head.	Students were willing and able to explain their thinking about how they solve problems.

Evidence of these themes can be seen throughout the pre- and post-assessment data. First, Student 8, who generally struggles with mathematics, was able to make a reasonable attempt at solving Assessment Question Number 2 (Deborah had some books. She went to the library and got three more books. Now she has eight books altogether.

How many books did she start with?) on the pre-assessment by asking me to reread the problem in chunks. Student 8 said, “Had some books. How many books?” at which time I reread the question. Student 8 then proceeded to lay out three books on the table, and then said, “She bought books and got three.” Student 8 then laid out more books on the table. Though Student 8’s final solution was incorrect for this problem, the attempted solution showed understanding of the individual steps needed to solve the problem. Student 8 also used a chunking strategy on the post-assessment. On Question 1 (Robin had four toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?), Student 8 said, “Can you read it again?” and after it being reread, Student 8 said, “Stop!” Student 8 then did the first step of the problem by making a rod of 11 unifix cubes. Afterwards, Student 8 said, “Can you read the problem again?” The problem was reread and after the second sentence Student 8 said, “Wait,” and proceeded to cover up four unifix cubes, the correct action for that part of the problem. Student 8 then touched and counted the remaining unifix cubes and came to the correct solution. Student 15 was able to correctly solve Pre-assessment Problem 4 (Some birds were sitting on a wire. Three birds flew away. There were eight birds still sitting on the wire. How many birds were sitting on the wire before the three birds flew away?) by dividing the problem into parts. After being read the problem, Student 15 mentally divided it into parts by first laying out eight birds. The student then said, “Some more came?” and laid out three more birds. Finally, the student touched and counted all of the birds and came to a correct answer.

Another theme that emerged from the pre-assessment and post-assessment was that students explained their thinking about how they solved the problems. On the pre-assessment, this occurred 30 times while the students were actively solving problems without being elicited with the prompt, "Tell me how you got your answer," after the students had stated their answers. Students did this 29 times on the post-assessment. Student 6 solved Pre-assessment Question 2 (Deborah had some books. She went to the library and got three more books. Now she has eight books altogether. How many books did she start with?) by explaining each step as it was done. After the problem was read, Student 6 said, "She got how many books," and set out three books. Student 6 then said, "She started with five books," and added five books to the pile of three. Student 6 went on to say, "I knew she got three more [books] so it equaled eight. 5 + 3 = 8." This was very similar to how Student 2 solved the same problem. Student 2 set out a row of eight books, and then explained, "So this is how much she has from the library." This student then counted and pointed to the five books on the end of the row, and said, "She got five more books from the library." Student 13 used the Number Facts strategy when explaining how to solve Post-assessment Question 1 (Robin had four toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?). Student 13 said, "I could do 4 + ? = 11. If I did 4 + 1 that'd equal 5. 4 + 6 that'd equal 10. If I did 4 + 7 that'd be 11. So I think I found my answer, 7. I know because 7 + 3 = 10 and one more is 11." After being prompted, Student 14 explained how to solve Post-assessment Question 3 (Roger had 13 stickers. He gave some to Colleen. He has 4 stickers left. How many stickers did he give to Colleen?) by saying, "Thirteen stickers. Nine. I put in my brain 13 [*sic*] and then I said that I know she had four so I took away four of them. I know 13 - 4 = 9. If you have 13 and you take away four you need to break the 10 into ones. You take four away. You have nine."

Analysis also revealed differences between students' pre- and post-assessment words and actions. Table 11 displays the themes, theme-related components, and assertions that were constructed from this assessment data.

Table 11

Pre- and Post-assessment Differences Themes, Theme Related Components, and Assertions

Themes	**Theme Related Components**	**Assertions**
Causes of incorrect solutions	Students made more guesses when answering pre-assessment questions (8) than when answering post-assessment questions (0). Students guessed the answer to a problem when they did not know how to solve it on the pre-assessment. Miscounting caused errors on the pre-assessment (13) but not on the post-assessment (0). The incorrect part of the number sentence was identified as the answer more on the pre-assessment (11) than on the post-assessment (0). Students used the incorrect operation to solve a problem more on the pre-assessment (14) than on the post-assessment (1).	There were four main reasons that students came to incorrect solutions on the pre-assessment.

(*table continues)*

Themes	Theme Related Components	Assertions
Strategies used to solve the problems	Many more answers were derived using number sentences on the post-assessment (53) than on the pre-assessment (11). Students more commonly used realia to help solve problems on the pre-assessment (57) than on the post-assessment (14). Students solved problems in their heads using visualization much more on the post-assessment (36) than on the pre-assessment (9). Students used fact families to assist in the solving of subtraction problems on the post-assessment (9). Students understood what the problem was saying and asking on the post-assessment.	As problem solving ability increased students used higher level strategies on the problem solving hierarchy, like visualization or number sentences.
Checking over work	A relatively small portion of assessment answers and solution steps were checked for accuracy (4 on pre-assessment and 22 on post-assessment). On the pre-assessment, all students who checked over their work had not made an error that they needed to fix (4). On the post-assessment, all students who checked over their work (22) found their error. Students checked their work by counting and recounting the manipulatives or realia they have used (4 on the pre-assessment and 9 on the post-assessment).	After the innovation, students tended to identify errors in their work when they considered the reasonableness of their answer.

(table continues)

Themes	Theme Related Components	Assertions
	Students justify their solution by stating a number sentence that could be used to solve the problem (11 on the pre-assessment and 53 on the post-assessment). Some students used multiple solution strategies when solving a problem to ensure accuracy of their answer.	

The differences in pre-assessment and post-assessment words and actions influenced the correctness of students' solutions. On the pre-assessment, students were more likely to guess at a solution strategy than they were to guess on the post-assessment. On the pre-assessment, eight answers were guesses made immediately after the problem was read without trying to first solve the problem and on the post-assessment no final answers were derived solely by guessing. This can be seen in Pre-assessment Question 1 (Robin had four toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?) when Student 3 gave the answer and the solution strategy by saying, "Ten. I just added in my head. Ten ones one [*sic*]," or when Student 3 answered Pre-assessment Question 2 (Deborah had some books. She went to the library and got three more books. Now she has eight books altogether. How many books did she start with?) by saying, "Three. I just guessed." Student 11 answered three out of the five pre-assessment questions with the answer of, "Some. I thought it in my mind." Other students, such as Student 16 tried a strategy and then gave up and said any number when solving the pre-assessment questions. This was more common when numbers in the problem exceeded 10, and students did not have the knowledge of those number facts yet and did not have enough fingers to use a Direct Modeling or Counting strategy when they were not using realia or manipulatives. Some students countered this by changing strategies, such as when Student 7 solved Post-assessment Question 3 (Roger had 13 stickers. He gave some to Colleen. He has four stickers left. How many stickers did he give to Colleen?) by first trying to solve the problem mentally and then changing strategies and drawing 13 stickers and erasing them. Student 7 then said, “This one I’m going to do base ten blocks but draw base ten blocks,” and proceeded to draw a rod worth 10 and three additional ones. Student 7 then said, “I think I can’t do base ten blocks. Wait, I’ll have to split the 10.” Then, lines were drawn on the rod to divide it into units. Student 7 then crossed off four of the units. Student 7 counted the remaining units and got nine. Then said, “No, I’m going to use dots. No, I’m going to use these [unifix cubes].” Student 7 made a rod of 13 unifix cubes and touched and recounted them aloud. Student 7 then pulled off four cubes from the end and touched and counted the remaining cubes. Student 7 gave the answer by saying, “Nine.”

Solution errors were also caused by students miscounting when using Direct Modeling or Counting strategies on the pre-assessment. Of the 95 total problems students solved on the pre-assessment, 13 answers were incorrect due in part to miscounting. On the post-assessment, no students’ miscounting caused an incorrect answer. On the pre-assessment, an error caused by miscounting happened when Student 11 solved Question 1 (Robin has four toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?) by counting incorrectly and making a rod of 10 unifix cubes rather than 11, which the problem called for. The student then said, “She had four,” and pulled four unifix cubes off the rod of 10. The student then touched and counted the six cubes left and said, “4 + 6 = 11. She had 11.” Students also miscounted when finding the final answer, as Student 19 did when solving the same problem. Student 19 set out four toy cars, and said, “You add.” Student 19 then counted on by laying out more toy cars, saying, “6, 7, 8, 9, 10, 11.” Student 19 counted the pile of toy cars that were just made and said the answer, “Six.” The miscounting by omitting the five in the second set caused the error.

Students also made errors on the pre-assessment by identifying the wrong part of the number sentence they created as the answer. On the pre-assessment, students thought that the answer of the number sentence was the answer to the word problem, regardless of where the variable was, 11 times. This did not happen after the implementation of the innovation on the post-assessment. On the pre-assessment, Student 4 answered Question 4 with the answer of eight birds, when the problem stated, “Some birds were sitting on a wire. Three birds flew away. There were eight birds still sitting on the wire. How many birds were sitting on the wire before the three birds flew away?” The correct answer was 11. This student solved the problem by laying out 11 birds in a row, and then pulling three birds away. Next the student drew three circles and eight squares on a piece of paper and recounted the squares. The student then wrote 11 - 3 = 8 birds. The student said, “First I added eight and three and then I took away three. Because you said some [*sic*]. I knew 11 - 3 = 8. I put the other birds on the tree as squares so I don’t [*sic*] get mixed up. I touched and counted each one. Eight.” Another type of number sentence error occurred multiple times on the pre-assessment as well. Students used the wrong operation when solving a problem with realia, manipulatives, a schematic representation, or in a number sentence on 14 problems on the pre-assessment as compared to only once on the post-assessment. Student 18 solved Pre-assessment Question 1 (Robin had four toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?) by setting out four toy cars, then counting out 11 more toys cars, and finally counting both piles one-by-one. Student 18 said that the answer was 15. This solution error was caused by adding and using the Joining All strategy rather than subtracting and using the Separating From strategy (Carpenter et al., 1999).

On the post-assessment, students employed more numerical and mental strategies to solve problems than they did on the pre-assessment, and on the pre-assessment students used more realia to aid in the problem solving process. Of other importance to this topic is that students were also able to understand what the word problem was saying and asking when completing the post-assessment. This was demonstrated when Student 10 said, “So we don’t know,” about the amount needed to start solving Post-assessment Question 2 which asked, “Deborah had some books. She went to the library and got three more books. Now she has eight books altogether. How many books did she start with?” This is quite different than the pre-assessment where Student 10 answered this question by saying, “She started with three,” and put out three fingers. Then continued by saying, “She bought four more,” and put out four more fingers on the other hand. Student 10 finally came to the conclusion of, “And she had eight.” Backing up this assertion, on the pre-assessment Student 11 answered questions with the response, “Some,” not knowing that questions were asking for a numerical value, whereas on the post-assessment, Student 11 gave correct numerical responses to all questions.

Higher level problem solving skills stood out on the post-assessment. Out of the 95 total problems students were asked to solve on the post-assessment, 53 of the problems were solved in part by using a number sentence. Students used number sentences as their first solution strategy and also to justify or check the answer they came up with using another strategy. Student 17 displayed how using a number sentence as a first solution strategy might look when solving Post-assessment Question 3 (Roger had 13 stickers. He gave some to Colleen. He has four stickers left. How many stickers did he give to Colleen?) by saying, "Nine cuz [*sic*] because 13 - 9 = 4." Another example of this was when Student 4 solved the same problem. This student wrote 13 - 9 = 4 stickers. The student went on to say, "He gave four to Colleen. I know because 9 + 4 = 13 and but [*sic*] I put 13 - 9 = 4 stickers because I just switched the numbers around." Student 2 showed how an answer could be checked using a number sentence when solving Post-assessment Problem 2 by describing that at first, "I put some in my head. Then I counted on until I got to eight." Student 2 modeled this step with head nods, and went on to say, "I know that 5 + 3 is 8. Because on the little slip it says 5 + 3 is 8." Additionally, 36 of the 95 post-assessment questions were solved mentally, without students using realia, manipulatives, or schematic representations, while only nine total pre-assessment problems were solved using strategies in students' heads. Student 1 demonstrated a mental solution strategy when solving Post-assessment Question 5 (Connie has 13 marbles. She has five more marbles than Juan. How many marbles does Juan have?) by saying, "Eight. First I tried doing 13 but I couldn't. I tried five and counted on to 13 and I got eight in my head." Six other students also used a mental strategy for this problem. Student 14's strategy was interesting and showed a deeper level of visualization. Student 14 said, "Eight. First I put 13 in a row in my brain. Then I put a line after five and counted the rest. It was eight." Student 19 showed knowledge of Recalled Facts when stating, “Five. Five. She has eight more than Juan!” This student went on to justify the answer by stating, “Juan has eight more marbles. 8 + 5 = 13.” Solving assessment problems using Number Facts was used extensively when the number facts were below 10 and involved facts students have been working with since kindergarten. Assessment Question 2 was solved by 14 out of the 19 participants by using a Number Facts strategy, generally Recalled Fact, with the students saying something similar to, “I know that 3 + 5 = 8.” Additionally, nine problems on the post-assessment used fact families, a variation of the Derived Facts strategy subset, to aid in solving subtraction number sentences, as opposed to only two problems having been solved using fact families on the pre-assessment. Student 9 solved Post-assessment Problem 4 (Some birds were sitting on a wire. Three birds flew away. There were eight birds still sitting on the wire. How many birds were sitting on the wire before the three birds flew away?) by first writing ? - 3 = 8 and then writing 11 - 3 = 8. When asked how this student solved this problem, the response was, “I know 8 + 3 in my head. It leaved [*sic*] me with 11. My answer is 11.”

On the pre-assessment students favored using realia to any other mode of problem solving. Fifty-seven of the 95 total problems solved during the pre-assessment were solved in part by using realia. This is opposed to only 14 of the post-assessment problems using realia in their solution strategy. On the pre-assessment, 12 out of the 19 answers on Problem 1 were solved using realia, 10 out of 19 on Problem 2, 10 out of 19 on Problem 3, 11 out of 19 on Problem 4, and 14 out of 19 on Problem 5. Problem 5, the problem comparing two people’s quantities of marbles, had the most students using realia. Only five students solved this problem correctly and three of them used the realia, which was marbles. Student 2 solved it correctly but differently than anyone else tried to solve it. Student 2 set out 13 marbles in a row. Then, this student set out more marbles in a row below until only the last five in the top row did not have a marble in the row below it. Student 2 made sure the marbles were matched up one-to-one with the marbles in the first row. Then this student said, “This one is a hard one,” and touched and counted the eight marbles in the bottom row. Student 2 then said the answer, “Eight.” The other students who used realia to help solve this problem tried to use Joining To, Separating From, or guessing as their strategy.

The final assertion I pose after examining the difference in pre- and post-assessment data is that when students checked over their work by evaluating the reasonableness of an answer, by recounting objects physically or mentally, or by doing the problem in two different ways, such as when students used a number sentence to check answers as stated earlier, they generally find the error they have inadvertently made. On the post-assessment, students did one of these forms of checking over their work on 22 of the 95 possible problems, and no errors remained. Students checked their work using recounting nine times on the post-assessment. An example of a student using recounting to check to make sure no errors were made happened when Student 5 was solving Post-assessment Problem 3 (Roger had 13 stickers. He gave some to Colleen. He has four stickers left. How many stickers did he give to Colleen?). This student counted out 13 stickers, then recounted the set aloud, and then touched and counted the set a third time. Student 6 did a similar recounting checking strategy when solving the same problem. Student 6 counted out 13 stickers mentally. Then Student 6 touched and recounted the set of stickers before proceeding to perform the rest of the actions in the problem. When something did not seem right when solving post-assessment problems, students were able to change strategies. Student 4 found an error by using multiple strategies when solving Post-assessment Question 1 (Robin had four toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?). Student 4 first said, "9 + 2 = 11. I added 4 + 5 and got 9. I knew 10 + 2 = 12 so 9 + 2 = 11." Student 4 then picked up a piece of paper and a pencil and drew four squares. Student 4 next drew and counted on seven more squares, and then wrote 4 + 7 = 11 toy cars. "Seven," Student 4 said and changed the solution to this problem, which was the correct answer. Other students besides Student 4 knew that their answers were incorrect and after unsuccessfully trying to justify the answers to themselves, they decided to use another strategy to solve the problem. For example, Student 5 answered Post-assessment Question 1 (Robin had 4 toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?) by first saying, "4 + 11 = ?," and then writing 4 + 11 = ?. Student 5 then drew four circles, and erased the circles. The student then drew four lines and 11 squares. Student 5 next crossed off three lines and crossed off 10 squares and wrote 4 + 11 = 2. Student 5 thought a while longer and then said, "Seven, because I know if you have 4 + 7 it equals 11."

Video recorded weekly problem solving observations. Three dyads were recorded weekly to gauge the effectiveness of the innovation, to find how students solve mathematical word problems, and to track students' progress through the problem solving hierarchy. The dyads were selected through rank order purposeful sampling and stratified random sampling of the class, showing three different levels of pre-innovation problem solving abilities, encompassing a low ability group, a medium ability group, and a high ability group. The weekly video recorded observations varied in length from 59 seconds to 4 minutes and 14 seconds. As Mack, Woodsong, MacQueen, Guest, and Namey (2005) suggest, I was able to benefit from participant observations by witnessing participants in the setting being studied and was able to get nuanced understandings by being with the participants during the event. This was critical to fully understanding the participants' experiences within the study and to being better equipped to make assertions about the study.

Actions and words of dyads were transcribed verbatim on the Video Recorded Observations Form. I then analyzed these observations and found some similarities and differences among dyads and noticeable, noteworthy changes that happened throughout the innovation period. Table 12 shows video recorded observations information, such as step in the innovation process, correctness of answer, number of words said, and length of observation, broken down by dyad.

Table 12

Video Recorded Dyads Observations Data Chart

	Dyad	Correctness	Words Said by Dyad	Length
Phase 1, Day 3	Low	No	80	3 min. 04 sec.
Compare, Difference Unknown	Medium	Yes	51	1 min. 33 sec.
	High	Yes	94	1 min. 36 sec.
Phase 1, Day 8	Low	No	120	3 min. 15 sec.
Part-part-whole, Part Unknown	Medium	No	144	4 min. 47 sec.
	High	Yes	181	3 min. 07 sec.
Phase 2, Day 3	Low	Yes	179	4 min. 20 sec.
Compare, Difference Unknown	Medium	Yes	60	1 min. 53 sec.
	High	Yes	110	2 min. 05 sec.
Phase 3, Day 3	Low	No	115	2 min. 44 sec.
Join, Start Unknown	Medium	Yes	103	1 min. 45 sec.
	High	Yes	107	2 min. 43 sec.

(table continues)

	Dyad	Correctness	Words Said by Dyad	Length
Phase 3, Day 8	Low	Yes	71	1 min. 58 sec.
Compare, Referent Unknown	Medium	Yes	102	3 min. 21 sec.
	High	Yes	181	3 min. 13 sec.
Phase 4, Day 2	Low	Yes	204	3 min. 10 sec.
Separate, Change Unknown	Medium	Yes	173	3 min. 01 sec.
	High	Yes	122	2 min. 25 sec.
Phase 5, Day 2	Low	Yes	55	0 min. 59 sec.
Separate, Results Unknown	Medium	Yes	58	1 min. 14 sec.
	High	Yes	92	2 min. 21 sec.
Phase 5, Day 7	Low	Yes	100	1 min. 39 sec.
Join, Change Unknown	Medium	Yes	116	1 min. 53 sec.
	High	Yes	117	2 min. 06 sec.
Phase 6, Day 2	Low	Yes	163	2 min. 36 sec.
Separate, Start Unknown	Medium	No	127	2 min. 36 sec.
	High	Yes	175	2 min. 33 sec.
Phase 6, Day 5	Low	Yes	71	1 min. 37 sec.
Join, Start Unknown	Medium	Yes	167	2 min. 44 sec.
	High	Yes	187	3 min. 35 sec.
Phase 7, Day 5	Low	No	97	4 min. 14 sec.
Part-part-whole, Part Unknown	Medium	Yes	75	1 min. 30 sec.
	High	Yes	114	3 min. 47 sec.
Phase 7, Day 10	Low	No	56	1 min. 43 sec.
Separate, Start Unknown	Medium	Yes	147	3 min. 02 sec.
	High	Yes	78	1 min. 48 sec.
Phase 7, Day 15	Low	No	47	2 min. 03 sec.
Compare, Compare Quantity Unknown	Medium	No	79	1 min. 17 sec.
	High	Yes	33	1 min. 00 sec.

Site-based interpretive research techniques, like the video recorded observations used in this study, are specifically beneficial in demonstrating what happens at one particular place, rather than across many places (Erickson, 1986). This type of fieldwork can describe the social action that is happening in the study (Erickson, Florio, & Buschman, 1980) and can be reported effectively though analytic narrative vignettes. Narrative vignettes allow the reader the vantage point of almost being in the research setting because well-written vignettes personify the analytical concepts and create a basis for readers to understand and believe what is being portrayed. Narrative vignettes are best created when the researcher is extremely thorough in noticing events in the study setting. Reflective descriptions in the form of vignettes can be effective in showing the everyday actions of the setting, as well as describing the major events that happened, and can be done from the viewpoint of the participants, the researcher, or an observer (Ely, Vinz, Downing, & Anzul, 1997; Erickson, 1986).

Vignettes encapsulate what the researcher has found in a digestible bite for the reader (Ely et al., 1997). Minor events are minimized or negated from the vignette, as to not muddy the waters and detract from the focus of the data transmission through the vignette. Vignettes are characterized as being easy to read and an effective way of portraying pages of field notes or narrative data. Contrary to some researchers' previous beliefs about vignettes as an untrustworthy instrument, when vignettes are built from deliberate analysis and facts, then they are trustworthy (Spalding & Phillips, 2007). The major drawback of vignettes occurs when the researcher does not portray a balanced description of what happened in the setting, or dwells on outlying situations (Erickson, 1986). Through careful qualitative analysis of the video recorded observation data, I have created descriptions of how students at three different levels would typically solve word problems. These vignettes portray a balanced, focused, and well-rounded depiction of daily problem solving throughout the study. The vignettes occur on two different days in this study. First, I will describe what would likely be seen from a low ability dyad, a medium ability dyad, and a high ability dyad when working to solve a CGI-style word problem during the students-at-work phase of the daily problem solving process at the beginning of the study. The problem the students will be working on takes place on the 13th day of the innovation, during Phase 2 on Day 3. The problem posed to students will be a Compare, Difference Unknown type problem. It is stated as, "Emma has eight blueberries and five grapes. How many more blueberries does Emma have than grapes?" Then, I will share a vignette that portrays a dyad in this study solving a Compare, Compare Quantity Unknown problem. This vignette will take place on the final day of the innovation, Day 60, which occurs in Phase 7 on Day 15. This problem is stated as "Peter has seven seashells. His friend Olivia has three more shells than Peter does. How many seashells does Olivia have?" Only one vignette will be used to portray typical problem solving behaviors of dyads for this problem because problem solving behaviors among video recorded dyads proved to be very similar at the end of the innovation period.

Vignette: Low ability dyad, Day 13 of innovation. Two lower ability students, Megan and Natalie, sit side by side at two desks. Before them are a small pile of blueberries and a small pile of grapes. Additionally, the dyad has a resealable baggie filled with colored unifix cubes at the top of Natalie's desk. Each girl has her pencil and answer recording slip. The day's problem has been read by the teacher and is posted on chart paper hanging on the whiteboard. Both students read the problem in unison, "Emma has eight blueberries and five grapes. How many more blueberries does Emma have than grapes?" Both girls sit for 15 seconds and look around. Megan laughs nervously. Natalie picks up a handful of the blueberries and starts counting them out loud. Megan sees Natalie doing this and starts picking up blueberries and handing them to Natalie. Natalie counts out 14 blueberries. Megan then starts counting the grapes out loud and finds that they have 10. “Ten,” Megan says as she tosses them down on the desk. Both girls look back up at the problem. Natalie says, “Emma has eight blueberries,” and pauses. She continues by saying, “We need to get eight blueberries.” Megan picks up blueberries and starts counting them one at a time, “1, 2, 3, 4, 5, 6, 7, 8.” “Stop!” Natalie yells. “We don’t need all of them.” “Why don’t we need all of them? We used all of them yesterday,” Megan asked. “We don’t need all of them because the problem says that she only has eight. We did it wrong yesterday. Remember?” Natalie explains. “Oh yeah,” Megan says quietly, not sure if she truly understands. Natalie picks up the grapes and reads the problem off the board, “Emma has eight blueberries and five grapes.” Natalie begins counting, “One, two, three.” Megan joins in and both girls continue to count, “Four, five.” “Stop,” Natalie says. Natalie moves the extra blueberries and grapes to the empty desk next to them. Both girls sit and look at the problem on the chart paper for 20 more seconds. Megan begins to read the problem again, “Emma has eight blueberries and five grapes. How many more blueberries does Emma have than grapes?” The girls wait silently five more seconds. Megan says, “Let’s count them altogether.” “Okay,” Natalie agrees. Megan picks up the grapes and begins counting them out loud to herself. At the same time Natalie picks up the blueberries and counts them aloud to herself. Megan says, “Five.” Natalie says, “Eight.” Both girls sit for another 10 seconds not saying or doing anything. Then Natalie picks up the blueberries and counts, “1, 2, 3, 4, 5, 6, 7, 8.” She then picks up the grapes and continues counting. This time Megan counts with her, “9, 10, 11, 12, 13.” Megan says, “13 more. Let’s write it down.” Both girls pick up their Answer Recording Slips and pencils and write 13 in the answer blank. Each girl’s Answer Recording Slip says “Emma has 13 more blueberries than grapes." Natalie says, "Let's do this using our cubes. I'll take out eight blue cubes and you take out the grapes." Natalie takes out eight blue cubes from the resealable bag and puts them in front of her. Megan took out five green cubes and put them in front of her. Both girls sit quietly for 5 seconds. The classroom is getting quiet at this time because all other groups are finished and students have returned to their seats and are silently reading, waiting for the other groups to finish. "Who wants to be the grapes?" Megan asks. "You be the grapes and I'll be the blueberries if we have to share our answer." Both girls get up and turn in their Answer Recording Slips. They return to their seats just as an MKO dyad is called to the document camera to share their solution strategy. This dyad spent 4 minutes and 5 seconds solving the problem and said 182 words during the students-at-work phase.

Vignette: Medium ability dyad, Day 13 of innovation. Two medium ability students, Andre and Sergio, sit side by side at two desks. In front of them are two Answer Recording Slips, one pencil, and a resealable bag of unifix cubes. There is a small pile of blueberries on Andre's desk and a small pile of grapes on Sergio's desk. Andre reads the problem written on the chart paper hanging on the white board at the front of the classroom. He reads the whole problem, "Emma has eight blueberries and five grapes. How many more blueberries does Emma have than grapes?" "We need to buddy them up," Sergio suggests. "Are you sure we buddy them?" Andre asks. "Yes, because we need to know how many more," Sergio replies. "You do the grapes and I'll do the blueberries." "Okay," Sergio says as he begins to line up grapes in a neat line across Andre's desk. Andre begins trying to lay out eight blueberries in a row above where Sergio is laying out the grapes, but their hands get in each other's way. Andre waits until Sergio gets done, then makes a neat row of eight blueberries above Sergio's five grapes. Andre says, "Now let's buddy them." Sergio takes his pencil and puts it on top of a set of one blueberry and one grape. Then he says, "Buddy." He moves the pencil to the next set and both boys say, "Buddy." They continue in the same fashion for three more sets of blueberries and grapes. Then there are three blueberries that have no matches. Andre says, "We have to count these. One, two, three," as he touches and counts the three blueberries. "Let me do it too. One, two, three," Sergio says. "Three more blueberries," Andre reaffirms. "Let's write it down." Both boys pick up their Answer Recording Slips. Sergio doesn't have his pencil and has to wait while Andre writes down his answer. "Can I borrow your pencil?" Sergio asks. Andre hands him his pencil and Sergio writes down his answer. "Who do you want to be?" Andre asks Sergio. "You be Emma," Sergio says, "and I'll hand you the blueberries and grapes." "Okay," says Andre. "Let's pretend. I'm Emma. Give me the blueberries and grapes," Andre says. Sergio hands Andre eight blue unifix cubes and five red unifix cubes. Andre silently lines them up in two neat rows. He then says, "Buddies, buddies, buddies, buddies, buddies," as he points to the pairs of blueberries and grapes. "Okay, we're done," Andre says. "Mrs. Spilde, can we eat the blueberries and grapes?" Sergio asks his teacher. Both boys get up and turn in their Answer Recording Slips and sit back down, take out their library books, and read silently. Total problem solving time is 1 minute 55 seconds with 113 words being said during the students-at-work phase.

Vignette: High ability dyad, Day 13 of innovation. Two high ability students, Annie and Zach, sit at desks next to each other. On Zach's desk is a pile of blueberries and a pile of grapes. On Annie's desk there are two Answer Recording Slips, two pencils, and a resealable baggie of unifix cubes. “Okay, okay, let’s read this,” Zach says frantically. Both students loudly start reading, “Emma has eight blueberries and five grapes. How many more blueberries does Emma have than grapes?” “Okay, let’s put them out,” Zach says. He starts laying out eight blueberries. “I’m going to be Emma, so give them to me,” Annie declares. Zach says, “Okay, lay them out,” as he hands Annie the blueberries. “1, 2, 3, 4, 5, 6, 7, 8, 9,” Zach says. “No wait, it’s supposed to be eight,” Annie says. “Oh yeah,” Zach says, agreeing that his partner caught his mistake. “Let’s count them again to be sure. 1, 2, 3, 4, 5, 6, 7, 8. Now put these in a neat row,” Annie says. “Now here are the grapes. 1, 2, 3, 4, 5,” Zach says as he hands the grapes to Annie one at a time. Annie lays them in a row below the blueberries. “Okay it says how many more blueberries than grapes does Emma have,” Zach says. “So let’s buddy them,” Annie says quickly and loudly. “Buddy, buddy,” Zach says as he points to the first two pairs of one blueberry and one grape. Annie joins in and both students say, “Buddy, buddy, buddy.” “Okay, there’re three left,” Zach says. “Emma has three more blueberries than grapes. Ha! You have three more blueberries than grapes,” Zach says. “Okay, now let’s do it with the cubes,” Annie says. Both students work to lay out the unifix cubes in the same fashion as the blueberries and grapes. Zach lays out eight green unifix cubes and Annie lays out the five red unifix cubes without saying anything. “Who’s going to talk?” Zach asks. “I will,” Annie says. “Let’s both talk,” Zach says, referring to if they are chosen to be the MKOs for the day and get to share their solution strategy with the class. “Let’s put our stuff away so we can be first,” Zach says. “Can we eat these?” Annie asks Zach. “Mrs. Spilde said after we are done with the problem solving then we can,” Zach said. Both students put their Answer Recording Slips on the table and return to their seats and begin reading. “Mrs. Spilde, can we share our answer?” Annie asks her teacher. This dyad completed the students-at-work phase in 2 minutes and 1 second and 144 words were used when working together to solve the problem.

Vignette: Typical dyad, Day 60 of innovation. A typical dyad, Jesus and Lacey, sits side by side in the classroom. The teacher has read the day’s problem, the class has restated the question, and dyads have spread out throughout the classroom. Both partners have their Problem Solving Journal in front of them and a pencil in their hand. “Okay, the problem says, Peter has seven seashells. His friend Olivia has three more shells than Peter. How many seashells does Olivia have?” Jesus reads to his partner, Lacey. Lacey says, “Okay, it says more so we have to subtract.” Jesus starts writing in his Problem Solving Journal and says, “We have to add because she has more.” He writes 7 + 3 = 10. Jesus says, “10,” and writes 10 in the answer blank. He then writes 7 + 3 = ? above the number sentence 7 + 3 = 10 he wrote previously. At the same time, Lacey says, “10 – 3 = 7,” and writes it on the line labeled *Number sentence* in her Problem Solving Journal. She then writes 10 on the answer blank line and writes the equation ? – 3 = 7 above the 10 – 3 = 7 number sentence she previously wrote. Lacey says, “I got ten.” Jesus looks up and says, “I got ten.” Both students continue looking at their own paper for about five more seconds and then Lacey says, “We’re done,” and closes her Problem Solving Journal and gets her library book out of her desk and reads. Jesus says, “Mrs. Spilde, can we share today?” The problem solving process took 58 seconds and the dyad said 54 words during the students-at-work phase.

Table 13 shows the dyads' highlighted problem solving traits portrayed in the previous problem solving vignettes.

Table 13

Problem Solving Traits by Dyad

	Beginning of Innovation Phase 2, Day 3
Dyad	Problem Solving Traits
Low	Spends periods of time sitting, not knowing what to do. Counts things together, handing realia and unifix cubes to partner. Relies mainly on adding, always uses Joining All strategy subset. Recognizes they only need the number of realia the problem stated. Still generally solves the problems incorrectly.
Medium	Shares information and strategies with each other, including reasoning. Divides up jobs to act out problems. Interacts with each other to solve problem. Asks questions about the problem to each other. Sits and looks around when they don't know what to do. Double checks their work.
High	Checks their actions with the words in the problem. Checks each other's actions and fixes if needed. Hands realia to each other to get the total needed for the problem. Corrects each other's mistakes. Double checks their work.

(table continues)

	End of the Innovation Phase 7, Day 15
Dyad	Problem Solving Traits
Low	Uses a ? for the unknown. Doesn't always agree on answer but doesn't talk about different answers. Says answer before writing the equation with the variable. Changes operation based on partner's work. Says number sentence right after reading the problem.
Medium	Talks through problem by putting it into math language/equation. Uses number sentence that doesn't exactly match actions of the problem. Checks over work by trying different numbers to see if answer is correct. Has discussion about answer. Writes equation with variable as last step, after number sentence written.
High	Changes number sentence to make it match the actions in the problem. Verbalizes number sentence and action in the problem. Checks reasonableness of the answer. Writes different number sentences but both worked. Solves problem independently.

The quantitative and qualitative data analyzed in this chapter will be used to create and warrant assertions and will be triangulated to provide information pertaining to the research questions guiding this study and create an overall picture of the findings of this study in the next chapter.

CHAPTER 5

FINDINGS

This study employed a concurrent component design, in that qualitative and quantitative data were collected throughout the study, remained separate through the collection and analysis process, and were not mixed until the interpretation and inference phases (Caracelli & Greene, 1997; Teddlie & Tashakkori, 2006). Through concurrent triangulation during the interpretation and inference phases, research methods were mixed and assertions were made and subsequently warranted (Creswell, 2009). Triangulation allowed for all data to weigh in on the same research topic–in the case of this study, the way students solve CGI-style mathematics word problems and the effect of the innovation–, as well as convergence in data to be sought, and reductions of study biases (Mathison, 1998). As an analyst, I brought my own biases, beliefs, thoughts, and experiences to the data analysis process, which is not necessarily a liability or an asset, but merely something that I acknowledged and was aware of when completing my written analysis (Corbin & Strauss, 2008). As suggested by Woolley (2009), by keeping an open mind to if the findings converged or diverged, and not being swayed by bias, I was able to develop a fuller use of the mixed methods framework that allowed for richer findings.

Procedures for Mixing Methods

Erickson's modified method of analytic induction was used to merge the quantitative and qualitative data in this study (Erickson, 1986). With the mixed methods purpose being triangulation, all data sources were weighted equally and were equally influential in the assertion process. All of the data and findings were read through, including traits and themes from the qualitative data, the Pre- and Post-assessment Student Answer Correctness Chart, the Daily Problem Solving Answer Chart, the Student Answer Solution Strategy Chart, the Video Recorded Observation Dyads Transcription Data Chart, t-test results, students' verbal and non-verbal solution steps from the pre-assessment and the post-assessment, and Video Recorded Observation Protocol transcription. Then the data were read through again, focusing on the interplay between the qualitative data assertions and themes and the quantitative data results. Sticky notes were used to record ideas, tentative assertions, and relationships among the data sources. Recording memos was an important strategy for keeping track of thoughts and ideas that were constructed from the data (Creswell, 2009; Johnson & Christensen, 2004). From these memos, a set of credible assertions was created based on ideas that were commonplace throughout the data. These tentative assertions were written as bullet points. A warranting process was conducted for each assertion by finding confirming and disconfirming evidence in the qualitative and quantitative data, acknowledging that warranted assertions are more reliable if confirming evidence comes from multiple data sources (Erickson, 1986). The goal of this process was not to prove what happened, but rather to show generalizable patterns within the data (Campbell, 1978). Genuine integration was desired in this study so counterpart analysis was necessary for addressing Research Question 2. This involved using both types of data to explore the same relationship between variables in the question, and was possible because I collected multiple data sources from the same instrument, my pre- and post-assessment (Yin, 2006). Based on the evidence found, unwarranted assertions were cast out or altered and credible final assertions were written. A presentation of the evidence was built to validate the assertions and is presented here. The study's research questions will be addressed after.

Warranted Assertions

Data were examined from several angles, starting from the general and working toward the specific (Creswell, 2009). Using Erickson's modified method of analytic induction, quantitative and qualitative data were combined to create assertions, which I subsequently warranted. During this process, I looked for confirming and disconfirming events and reported on both to test the evidentiary warrants for my assertions, as well as key linkages so that strong bonds were made between data sources and events occurring in the study (Erickson, 1986).

Assertion 1: Students' problem solving abilities increased from participating in daily CGI-style word problem solving through guided incremental steps. As stated earlier, this study defines problem solving abilities as the accuracy of solutions, the speed at which problems are solved, the solution strategy used to solve the problem, the understanding of the problem, and the understanding of the solution strategy. Evidence used to warrant this assertion combines quantitative pre-assessment and post-assessment data, quantitative daily answer correctness data, quantitative daily solution length, qualitative pre-assessment and post-assessment data, and qualitative video recorded dyads data.

Correctness of student answers on pre-assessment to post-assessment. All 19 students who participated in this study increased the number of problems they solved correctly on the post-assessment as compared to their pre-assessment scores. Increases ranged from 20.00% to 100.00%, with three participants increasing their score 20.00%, three increasing their score 40.00%, six increasing their score 60.00%, two increasing their score 80.00%, and five increasing their score 100.00%. These increases were statistically significant ($p < .001$). The mean pre-assessment score was 33.68% correct, whereas the mean post-assessment score was 96.84% correct, an increase of 63.16% from the pre- to post-. A paired-samples t-test indicated that this increase in student problem solving performance can be associated with the innovation rather than occurring by chance.

Correctness of student answers on first third of daily problem solving compared to last third of daily problem solving problems. The average percent correct on the first 20 daily problem solving questions was 75.01% and the average percent correct on the last 20 daily problem solving questions was 85.34%. This was an increase of 10.33%, which was found to be statistically significant ($p < .001$). This result shows that the increase in student daily problem solving correctness can be associated with the innovation rather than occurring by chance.

Time spent solving daily problems by dyads at beginning compared to end of innovation. The amount of time spent solving daily word problems decreased overall throughout the innovation implementation period. A comparison was made between the mean students-at-work length during Phase 1 at the beginning of the innovation with the mean students-at-work length during Phase 7 at the end of the innovation. A Phase 1 problem took an average of 2 minutes and 54 seconds to solve and a Phase 7 problem took 2 minutes and 16 seconds. This was a decrease in time spent solving problems of 38 seconds, 21.84%. Though this difference was not statistically significant, it shows an improvement in the efficiency of students' problem solving process.

Higher level solution strategies used on post-assessment than pre-assessment. Overall, the solution strategies students used to solve questions on the post-assessment were at a higher level of complexity than the solution strategies students employed on the pre-assessment. From the pre-assessment to the post-assessment, there was a decrease from 8.42% to 0.00% of the problems being solved by no solution strategy or a guess, a decrease from 73.68% to 37.89% in the problems being solved by a Direct Modeling strategy, an increase from 6.32% to 15.79% in problems being solved by a Counting strategy, and an increase from 11.58% to 46.32% in the problems being solved by a Number Facts strategy. This shows an overall shift from the use of lower level solution strategies to an increased use of higher level solution strategies. In fact, Direct Modeling was stated as one of the most common solution strategies used on all five of the pre-assessment questions, whereas Number Facts was stated as one of the most common solution strategies on the post-assessment for four out of the five questions. Additionally, when solution strategy subsets were inspected, the same trend held true. The greatest decrease in solution strategy subset usage from the pre-assessment to the post-assessment was in Direct Modeling, Separating From, which showed a decreased in occurrence of 10.52%. The greatest increase was in Number Facts, Recalled Fact, which showed a gain in occurrence of 27.37%.

Guessing immediately on pre-assessment versus post-assessment. Students on the pre-assessment guessed almost immediately on 8.42% of the problems. This was eight out of the 95 total answers students gave. On the post-assessment, no student gave an immediate guess answer. This was a decrease of 100.00% in the number of guesses. For example, Student 3 showed development in effort put into answers. Student 3 answered Pre-assessment Question 3 (Roger had 13 stickers. He gave some to Colleen. He has four stickers left. How many stickers did he give to Colleen?) by guessing and nearly immediately saying, “Twelve. I remembered it from the story.” Then on the post-assessment, Student 3 answered the same question by counting out 13 stickers aloud, moving away nine stickers, counting one-by-one until four stickers were left in the pile, and then saying the answer, “Nine.”

Understanding of parts of equations. As previously illustrated in the differences between the pre- and post-assessment themes, students used number sentences often when describing their post-assessment solution strategies. All 53 times a student stated a number sentence on the post-assessment, it was associated with a correct answer. On the pre-assessment, number sentences were only stated 11 times, and four times they lead to an incorrect solution. Additionally, students identified the incorrect part of the number sentence or the incorrect portion of the manipulatives, realia, or schematic representation in 11 of the problems on the pre-assessment. An example of this was when Student 4 wrote the correct number sentence 3 + 5 = 8 for Pre-assessment Question 2 (Deborah had some books. She went to the library and got three more books. Now she has eight books altogether. How many books did she start with?), but then identified the 8 as the answer, when the addend 5 was the correct answer in this Join, Start Unknown problem. Students made these types of errors zero times on the post-assessment.

Threats to validity. Validity must be considered when warranting an assertion. Two threats to internal validity, history and maturity, could possibly be factors affecting students’ problem solving abilities, and therefore this assertion. First, when designing this study and writing additional lesson plans not associated with this project, I considered the effect of development that normally occurs in my second grade classroom. Because of this, I used this daily problem innovation as my students' primary form of mathematics problem solving instruction. Additional problem solving questions were kept to a minimum and to topics other than the basic addition and subtraction CGI-style word problems that were the focus of this study. When considering the effects of normal maturation, the students in this study far surpassed the problem solving abilities of previous students at this point in the school year. This was seen in the pilot of the pre-/post-assessment that took place in the Spring of 2012. The students in this study outscored the students who piloted the assessment but who did not receive the innovation.

Assertion 2: Students internalized the solution strategy process by participating in this innovation. Effective problem solvers can unpack a problem and visualize its steps (Hegarty et al., 1995). Internalization of the solution strategy can be seen by students solving the problem in their head or using fewer aides, such as realia, to solve the problem (Montague, n.d.). Evidence to warrant this assertion includes the increase in the use of the Number Facts strategy, decrease in the prevalence of lower complexity solution strategies, a reduction in the reliance on the aide of realia to solve problems, an increase in the immediacy of answers, and the increase in the necessity of probing questions to elicit students' solution strategies.

Number Facts usage increased on post-assessment. When solving the post-assessment, students tended to state a number sentence along with their answer. This occurred 53 times on the post-assessment. In comparison, on the pre-assessment only 11 students stated a number sentence. As found in the paired-samples t-test results stated earlier, the shift from simpler to more complex solution strategies from the pre-assessment to the post-assessment was statistically significant. Highlights of these findings were that the Direct Modeling strategy subsets were used in 73.68% of the pre-assessment solutions but only 37.89% of the post-assessment strategies. This was a decrease of 35.79%. On the other hand, Number Facts strategy subsets were used 11.58% on the pre-assessment and in 46.32% of the answers on the post-assessment. This was an increase of 34.74%. In fact, Number Facts, Recalled Fact was one of the most common solution strategy subsets used on all post-assessment questions, whereas, Direct Modeling strategy subsets were one of the most common solution strategies on each of the pre-assessment questions. Additionally, the use of fact families, a derivation of Number Facts, Derived Fact, was seen nine times on the post-assessment, and only twice on the pre-assessment.

Less use of realia on post-assessment. As discussed earlier, the solution strategy and strategy subset used by students on the pre-assessment and post-assessment changed due to this innovation. The use of realia can be associated with lower level solution strategies, such as Direct Modeling and Counting strategies and their strategy subsets. This innovation was designed to introduce problem solving through the use of realia to build understanding in the actions and operations needed to solve problems more efficiently in the future. The innovation plan was designed to gradually decrease the use of and dependence on realia to solve problems, but maintain understanding when solving problems in more complex manners. This plan proved successful. On the pre-assessment, students used realia to aide in solving a problem 57 times. On the post-assessment, this number had dropped to 14 times throughout the entire assessment. Additionally, the Direct Modeling strategies associated with the use of realia decreased a statistically significant amount. For example, Student 17 solved Pre-assessment Question 3 (Roger had 13 stickers. He gave some to Colleen. He has four stickers left. How many stickers did he give to Colleen?) using realia by first counting out a set of 13 stickers, then setting four to the side, and finally counting the pile left one-by-one. This same student solved Question 3 on the post-assessment using a Number Facts strategy by saying, “Nine. Cuz [*sic*] 13 - 9 = 4.” Student 12 showed an alternative way to solve this problem using visualization. This student imagined 13 stickers and then mentally took away four of them and came to the correct answer. On the pre-assessment this student solved the same problem using a Counting strategy.

Said answer immediately. A difference in the ease with which students came to answers could be seen when comparing the pre-assessment to the post-assessment. On the post-assessment students tended to give a correct answer nearly immediately after the problem was stated. On the post-assessment students stated the correct answer immediately 24 times, whereas on the pre-assessment, students stated an answer immediately 13 times, but only five of these immediate answers were correct. The rest were immediate guesses.

Shared thinking when questioned on post-assessment. Students tended to talk equally during the pre-assessment and post-assessment, but the way students explained their solutions varied by assessment. Students tended to explain how they solved a problem in real-time, while they were figuring out their answers, more often when the problem was not solved mentally. Conversely, students tended to need prompting on the post-assessment to explain how they solved the problem. This was accomplished by asking students to explain how they solved the problem after the answer was given. This coincides with earlier findings that students more commonly gave an immediate answer when solving problems on the post-assessment than on the pre-assessment, and their solution strategy complexity increased at a significant level from the pre-assessment to the post-assessment. Overall, students needed to be prompted to explain their problem solving process 15 times on the pre-assessment and 26 times on the post-assessment.

Threats to validity. The experimenter effect, history, and maturation could be threats to validity in this study and were considered when warranting this assertion. First, the experimenter effect was countered through the way the pre- and post-assessment data collection was designed. All student actions and words used when solving a problem were recorded, along with any clarifying statements about solution strategies that students used after the problem had been solved. Students' solution strategies and answers were transcribed and analyzed after all students had been assessed. The chance of assessment data being skewed by the research method or researcher was minimized by this process. Additionally, history and maturity could have played factors in the results, so they were considered when designing the study and analyzing data as well. Students generally gain knowledge as the school year progresses, so to encourage students developing their mathematics skills without influencing the results of this study, topics other than mathematics problem solving were taught during daily mathematics lessons. Further, the use of MKOs and strategy conferences to discuss problem solving strategies were only used during the daily innovation time, which limits the idea that additional students sharing higher solution strategies attributed to students' increase in internalization of solution processes.

Assertion 3: Students worked more independently on problems as their problem solving abilities increased. Students worked with their partner when the innovation began. The innovation was designed so that like-ability dyads would begin the problem solving process by acting out problems together, building understanding through their actions. Throughout the innovation, the process was designed to continue interactions between dyad members, but in a different way. As the innovation progressed, students were not to rely on their partner to help act out problems, but rather to share ideas with. Evidence to warrant this assertion includes the number of words dyads said during their students-at-work portion of the lesson, the reduction in problem solving session lengths, student problem solving traits, and the increase in correctness of daily problem solving questions.

Number of words said during daily problem solving students-at-work portion. As shown through the vignettes portrayed in Chapter 4, students interacted with each other much more during the beginning of the innovation. During Phase 1 of the innovation's students-at-work phase of the daily problem solving process, the three dyads averaged 111.67 words said to each other. During Phase 7 of the innovation, the average number of words dyads said during the students-at-work phase was 80.67 words. This decrease in the number of words said was a sizeable decrease of 27.76%, but a t-test showed that the decrease could not confidently be related solely to the innovation because the results were not statistically significant. This finding was likely due to the small variance in words said by the medium ability dyad and small sample size.

Shorter problem solving times during students-at-work portion. The dyads' students-at-work portions of the daily problem solving session were recorded once weekly. It was found that the average length of time students spent solving the problem decreased over the course of the innovation. The mean problem solving length of Phase 1 of the innovation (2 minutes and 54 seconds) was compared to Phase 7 of the innovation (2 minutes and 16 seconds). Although the findings were not statistically significant, likely due to small sample size, they were still notable and of interest. All dyads decreased their average problem solving lengths from Phase 1 to Phase 7, with an average decrease of 21.84%.

Student problem solving traits. Video recorded observations and the transcription of students solving the pre-assessment and post-assessment showed that students were able to access problems more readily as the innovation progressed. As stated earlier, students immediately guessed on eight pre-assessment questions, but did not immediately state a guess without first trying a strategy on any post-assessment questions. Additionally, video recorded observations showed that all three of the dyads readily looked at each other, paused in their problem solving waiting for their partner to tell them what to do, and stared around the classroom without working on the problem at the beginning of the innovation period. This non-working period persisted longer into the innovation implementation for the lowest ability dyad than the other dyads. At the end of the innovation period, students in all three dyads were attempting to solve problems immediately and long periods without working on the problem were not evidenced.

Correctness of daily problem solving questions. The correctness of daily problem solving answers increased as a result of the innovation. There was a statistically significant increase in the correctness of the last 20 daily problem solving questions as compared to the first 20 daily problem solving questions. During the first third of the innovation, 75.01% of the answers were correct. During the final third of the innovation, 85.34% of the answers were correct. This increase was 10.33% more correct. Viewing these findings alongside the findings of a decrease in problem solving times and number of words creates an interesting relationship between correctness, independence, and efficiency.

Threats to validity. When warranting this assertion, one primary threat to validity, the novelty effect, was considered. Because this innovation was 14 weeks long, and students solved a word problem each day, I was aware of the effects I could have on students' overall problem solving process, especially the students-at-work and strategy conference portions of the daily problem solving routine. To ensure my actions did not affect results in problem solving independence, I worked to behave in the same manner during every day of the innovation. I strove to provide a relaxed, unrushed classroom environment where students felt comfortable to take their time solving problems and had plenty of opportunities to share their thinking and ask classmates questions about their solution strategies.

Assertion 4: Students checked over work more frequently as a result of participation in this innovation. Students grew in many different and unexpected ways by participating in this innovation. One of those ways was their increase in understanding of word problems that led to their propensity to check over their work and find errors. Data used to warrant this assertion includes a comparison of the checking actions during the assessments, students' increase in trying multiple solution strategies, and students' words and actions when solutions were not correct.

Found errors on post-assessment more often than pre-assessment. Students increased their problem solving abilities as a result of this innovation, as evidenced through a t-test comparing pre-assessment to post-assessment scores. Part of the increase in assessment scores was attributed to students' ability to identify errors in their solution strategies and fix them. On the pre-assessment, students checked over their work four times, but on the post-assessment students checked their work 22 times. On the pre-assessment, all students who checked over their work had not made an error. What is more notable about these statistics is that when students employed this checking strategy they found all of their errors on the post-assessment. Recounting of the manipulatives used to solve the problem occurred four times in the pre-assessment and nine times on the post-assessment.

Justified answers with number sentences. The Number Facts strategy, including number sentences, is the highest complexity level that students used to solve problems in this study. Through this innovation, students showed dramatic growth in their use of number sentences to justify their answer. On the pre-assessment, students used number sentences 11 times and on the post-assessment students used number sentences 53 times. Number sentences were most commonly stated after the answer was given, but they were also used as part of the students' solution strategies when talking through a word problem.

Knew when answer did not make sense. Students exhibited their discomfort with their incorrect answer when solving problems. Most commonly, students changed solution strategies when they realized that the answer they had given was incorrect and it could not be justified. As an example, one student started solving Post-assessment

Question 3 (Roger had 13 stickers. He gave some to Colleen. He has four stickers left. How many stickers did he give to Colleen?) and then paused and changed solution strategies. The student first wrote 13 - ? = 4 on a piece of paper. Then said, "Uh. 6. I thought in my head." Next, the student started at 13 and counted backward, saying, "12, 11, 10, 9, 8, 7," and put up six fingers and then took away two more. The student wrote 13 - 8 = 4, put up fingers, and counted backward eight numbers, "13, 12, 11, 10, 9, 8, 7, 6, 5." The student then drew 13 circles on the paper and counted aloud as they were drawn. Next, the student said, "13 - some = 4," and proceeded to leave the first four circles on the paper and cross off the last 9 circles. The student then circled the first four circles and said, "Nine, I mean." This is in comparison to how this student solved the same pre-assessment question. On the pre-assessment, the student counted out a pile of 13 stickers one-by-one and then recounted the pile one-by-one. The student then counted a separate pile of four stickers one-by-one and finally counted all of the stickers together and said, "17." This student did not realize that the answer on the pre-assessment was wrong and did not make mathematical or logical sense. On the post-assessment, the student persevered until the answer made sense, trying multiple strategies until a correct solution was found. A total of 11 students tried multiple solution strategies on the post-assessment, and 10 of these 11 students eventually came to the correct solution.

Threats to validity. Validity was considered when warranting this assertion. Care was taken to ensure that the experimenter effect did not influence results in this study, because if I, as the researcher and practitioner, changed my affect, words, or tone while a student was giving an answer, the student could be clued in to whether that answer was correct or not, and this could influence this assertion. I was aware of this while assessing students, and while I was transcribing data. I paid close attention to the wording I used while asking students to explain their thinking so that all students received the same treatment from me regardless of the correctness of their answer. When reviewing the written description on the pre-assessment and post-assessment Student Answer Recording Forms, it was evident in the description of student assessment solution strategies and answers that this did not impact student answers.

Research Questions

Quantitative data and qualitative data were triangulated to shed light on the two research questions that guided this study. Through triangulation, multiple data sources weighed in on one topic, creating a clearer picture of the situation (Gay et al., 2009). This triangulated data will be viewed through the theoretical lenses that guided the design of the study, including Vygotsky's social development theory, Bandura's social learning theory, and Piaget's and Vygotsky's theories of constructivism.

Question 1: How does a class of second grade students at San Marcos Elementary solve Cognitively Guided Instruction-style contextual word problems?

Quantitative and qualitative findings from the pre-assessment, quantitative and qualitative findings from the post-assessment, and quantitative and qualitative findings from the video recorded observations were joined to weigh in on this question. The triangulated results from this study showed that students in this class solve CGI-style word problems correctly, with understanding at a high complexity level, and cooperatively with developed independence.

As evidenced by the pre- and post-assessment results combined with daily problem solving correctness and video recorded dyads' problem solving times, students in this class showed they preferred to use higher level solution strategies to solve CGI-style word problems. Prior to the innovation, students most commonly used Direct Modeling strategies to solve mathematics word problems, with 73.68% of all problems on the pre-assessment being answered with this lower complexity solution strategy. Also on the pre-assessment, 8.42% of questions were answered when students did not use any strategy and immediately guessed on answers, 6.32% of questions were answered using a Counting strategy, and 11.58% of questions were answered using a Number Facts strategy. Students were scaffolded through the problem solving hierarchy during this innovation and students' problem solving strategy complexity increased as a result. On the post-assessment, the Number Facts strategy was the most common strategy used, with 46.32% of questions being answered using this strategy. This was an increase in the use of the Number Facts strategy from the pre-assessment to the post-assessment of 34.73%. Additionally, on the post-assessment, students decreased the usage of Direct Modeling strategies by 35.79%. Counting strategies increased from 6.32% usage on the pre-assessment to 15.79% on the post-assessment, a gain of 9.47%.

On the daily problem solving questions, students increased their correctness from the beginning of the innovation, where lower complexity solution strategies were used, to the end of the innovation, where higher level solution strategies were used to solve daily word problems. When comparing the first 20 daily problem solving questions to the last 20 daily problem solving questions, there was a statistically significant increase in the class's mean correct between the two sets. In addition, students' problem solving times decreased throughout the innovation period. Onslow (1991) states that students who use higher complexity solution strategies to solve word problems generally solve the problem more efficiently and quickly. This coincides with what participants in this study showed.

Prior to the innovation, most members of the class were missing the conceptual understanding of actions implied by word problems and their related mathematical functions. On the pre-assessment, students showed this lack of understanding by guessing at answers (8.42% of answers) and performing the incorrect operations to solve problems (14.74% of answers). These incorrect operations usually took the form of number grabbing or subtracting when the problem should have been solved by joining the numbers, which coincides with what Peter-Koop (2005) describes as what happens when a student does not comprehend the wording of a problem or its mathematical basis. This lack of understanding of the problems led to a pre-assessment class average of 33.68% correct. After the innovation, students showed understanding of the operations and actions needed to solve word problems, and used a strategy they felt comfortable with to solve the problem, resulting in a class average of 96.84% correct on the post-assessment. Though not every student could solve every problem, there was a dramatic increase of 63.16% from the pre-assessment to post-assessment, as well as the increase in solution strategy complexity stated previously. Both of these increases were statistically significant. Because of this innovation, more students in this class are able to solve more problems correctly. This coincides with the findings of Arzarello et al. (2005) in their study involving the use of realia and bodily movements to foster understanding in problem solving and increase the problem solving abilities of a group of intermediate elementary students.

The flexibility which allowed students to choose which solution strategy to use to solve problems on the assessments proved to be beneficial to students' overall problem solving abilities. The design of the innovation involved dictating the solution strategy students could use to solve daily word problems, with an overall increase in complexity of strategies students would use over the course of the innovation. This showed to be an effective design element in regards to overall problem solving ability, but an interesting result was that students fared better on the post-assessment than they did while solving daily word problems. This was likely because students could use any strategy, realia, or manipulative they needed to solve each word problem on the post-assessment. Whereas, on the daily problem solving questions, students were directed as to which strategy they could use to solve that problem. Students scored 96.84% correct on the post-assessment and 85.34% correct on the final third of the daily problem solving problems. This finding coincides with what CGI suggests–that students benefit from being allowed the freedom to solve a word problem in any way that makes sense to them (Carpenter et al., 1999)–and what social learning theory demonstrates through its description that students watch others and the outcomes they obtain, and then decide what they fully understand and will use as their own methods (Bandura, 1977).

Students in this study also solved CGI-style word problems with understanding. This was evidenced by the words and actions students used to solve the pre- and post-assessments, video recorded dyads' words and actions, and video recorded dyads' problem solving times. As asserted previously, students checked over their work more on the post-assessment (22 times) than they did on the pre-assessment (four times), and this led to more correct answers, as well as demonstrated that students had an understanding of when their answers were correct or incorrect. When students were confronted with an answer that they could not justify using a number sentence or an answer that they did not believe was correct, an additional solution strategy was often tried on the post-assessment. Ten out of the 11 times students tried multiple solution strategies, they were able to come to a correct solution. This showed that students understood what they were doing to solve a problem, could use strategies flexibly, and understood the reasonableness of their answer when solving CGI-style word problems with single-digit and lower two-digit numbers. Additionally, on the post-assessment, more students were able to come to the correct answer immediately, then justify their work with an explanation of how they came to that answer or a number sentence that they used to solve the problem. On the pre-assessment, students also described how they solved the problem, but this description most commonly was done as students worked their way through the problem solving process. Correct answers were less commonly immediately said on the pre-assessment (five times) than on the post-assessment (24 times).

When students worked with their partner, they began by working more dependently with their partner, relying on the other's knowledge to aide in the problem solving process, as evidenced in the video recorded observations. Toward the end of the innovation, most dyads were working more independently to solve problems and no longer relied as much on their partners' support to answer questions. Partners were able to solve problems correctly on their own and justify their work, commonly with a number sentence or schematic representation. Video recorded observations also showed that students were able to work well with their partner when needed. They shared their thinking with their partner when they wanted to, which happened more at the beginning of the innovation, and solved the problem independently as they became more adept at problem solving, which more commonly occurred toward the end of the innovation. Students willingly shared their thinking with their partner when asked, and stated their answer out loud when solving the problem independently. When students were confident in their answer, they chose not to listen to their partner's solution and chose their own solution as their final answer. Though the progression of the problem solving strategies employed by this innovation may have contributed to this shift in students' problem solving independence, the flexibility demonstrated by students coincides with what many other researchers have found in their studies of like-ability versus mixed-ability dyads (Denessen et al., 2008; Schmitz & Winskel, 2008; Takako, 2010). Generally, they found that like-ability dyads had the propensity to create greater understanding and skill than mixed-ability dyads. Additionally, students also decreased their problem solving times and number of words said during the students-at-work portion of the daily problem solving process overall from the beginning of the innovation to the end of the innovation, and with understanding comes efficiency and flexibility (Onslow, 1991).

Students in this classroom not only worked cooperatively with their partner, but also worked cooperatively as a class. A major theoretical focus of this study was designed around the benefits of the MKO. This study was designed to help students develop their mathematical understandings by listening to and questioning the MKO during the daily strategy conference, a form of scaffolding to help raise students' ZPDs (Vygotsky, 1978). Each day, a dyad or myself shared a successful, and usually more complex solution strategy with the class. This played a part in many observable advancements in the classroom. The lowest ability dyad learned that they did not need to use all of the realia or manipulatives they were given to answer a question. On the third video recorded observation, this dyad showed for the first time that they understood that they only needed to use what the problem said, rather than trying to use all of the items they were given to solve the problem. Growth in this area may not have come without modeling by a more capable classmate. Additionally, the medium ability dyad began stating and writing a number sentence to solve each problem in the video recorded observations during Phase 4, Day 2. This was before Phase 6 of the innovation when number sentences were to be formally introduced. This caused other students in the class to try to add number sentences to their schematic representations. The high-ability dyad first showed use of a written number sentence on Phase 5, Day 7, the low ability dyad first used a written number sentence on Phase 5, Day 2, and the majority of the class was including number sentences in their problem solving journals before Phase 6 officially began. Though the effects of the MKO were not directly calculated in this study, its effects can still be seen in solution strategies and answer correctness. The effects of the MKO are supported by social development theory and social learning theory. Evidenced in the increase of average percent correct from the beginning third of daily problem solving questions to the final third of the problem solving questions, students developed their ability to correctly solve mathematics word problems. Without seeing others solve problems, hearing their explanations, and talking about their own mathematical work, it is reasonable to assume that problem solving abilities would not have improved a significant amount, just as Cloutier and Goldschmid (1978) found. Students in this class watched others and then made up their own understandings, ideas, and beliefs about how to solve different types of CGI-style word problems. As evidenced throughout the study, student participants developed at different rates, and solution complexity was not at the highest level for all students at the end of the innovation for all problem types. Not all students used the highest strategy complexity to solve all problems, though many did as a result of this innovation, and this overall increase in solution strategy complexity was found to be statistically significant. On the post-assessment, 37.89% of the questions were still answered using Direct Modeling, including realia, manipulatives, and drawings, but these aides now had meaning for students. A strong elementary school mathematics classroom combines concept developing, quality mathematics activities, student conversation, and opportunities for students to build their own understandings of mathematical concepts (Carpenter et al., 1999; Kilpatrick & Swafford, 2002; Kline, 2008; National Research Council, 1989; NCTM, 2000; Sutton & Krueger, 2002). Through this innovation, students were able to work at their own pace, in like-ability dyads to construct meaning within problems (Piaget, 1953; Vygotsky, 1962), and the use of the MKO to share solution strategies with the class scaffolded students with lower bottoms to their ZPDS (Vygotsky, 1978).

Question 2: How and to what extent does partnered Cognitively Guided Instruction-style mathematics word problem solving through guided incremental steps affect a class of San Marcos second graders' mathematics problem solving abilities? Quantitative and qualitative findings from the pre-assessment, quantitative and qualitative findings from the post-assessment, quantitative findings from daily problem solving answers, and quantitative and qualitative findings from the video recorded observations will be joined to weigh in on this question. The triangulated results from this study showed that students in this class increased their problem solving abilities, awareness of parts of number sentences, understanding of the reasonableness of answers, mental actions in problem solving, efficiency, and efficacy.

Participation in this study positively impacted students' problem solving abilities. From the pre-assessment to the post-assessment, the class mean correct increased 63.16%. This increase was statistically significant so this increase in correctness was likely was not attributed to chance but rather to the innovation itself. Additionally, students' daily problem solving responses increased 10.33% in mean correctness throughout the innovation, which was found to be statistically significant using a paired-samples t-test. Triangulating this with students' increase in complexity in problem solving solution strategies and decrease in the length of time video recorded dyads took to solve problems creates a converged picture of increased problem solving ability that can be directly related to the innovation. This increase was more than would be expected through the history effect, because when students piloted the assessment in the Spring of 2012, students did not score at nearly 100% correct, as they did on the post-assessment in this study, even though the primary mathematics instruction for the two groups of students was nearly identical.

Through participation in this study, student developed an increased awareness of the meaning of the parts of number sentences. On the pre-assessment, students selected the wrong part of a number sentence as the answer 11.58% of the time. By comparing this to the post-assessment, where students did not choose the wrong part of the number sentence as the answer to the problem any times, growth from participation can be seen. Further, students used number sentences to solve problems on the post-assessment and to justify their answers 53 times on the post-assessment. Understanding of the parts of a number sentence and actions indicated by symbols in the number sentence is generally necessary to come to a correct solution (Onslow, 1991), especially at the rate that students did on this assessment.

Students in this study showed an increase in their ability to check over their work and answers and in their awareness of the reasonableness of their answers. This can be evidenced through the number of times students checked their work on the pre-assessment (four times) compared to the number of times work was checked on the post-assessment (22 times.) Further, students did not settle for incorrect answers on all but three questions on the post-assessment. When a solution or strategy did not seem to make sense to a student or an answer couldn't be justified, students tried other strategies or redid their work until a reasonable solution was found.

Students increased their abilities to visualize a problem and use mental strategies to solve it through participating in this study. From the pre-assessment to the post-assessment, students showed an increase of 34.73% in the usage of the Number Facts strategy to solve problems. There was also a decrease of 35.79% in the number of problems solved using a Direct Modeling strategy. Student 6 demonstrated this decrease when the student solved Pre-assessment Question 5 (Connie has 13 marbles. She has five more marbles than Juan. How many marbles does Juan have?) by using Direct Modeling, Joining All and the same question on the post-assessment by Number Facts, Recalled Fact. These changes in class solution strategies were shown to be statistically significant using a paired-samples t-test, so with confidence, it can be said that the innovation affected this change. Students also decreased their overall use of realia from 57 times on the pre-assessment to 14 times on the post-assessment. The study was designed to use realia to help build understanding in the words and actions of problems and the problem solving process, which it did. Many students realized that the use of realia was not the most efficient way to solve problems and chose not to use it when it was not necessary because a more efficient solution strategy was able to be used. This coincides with Englard's (2010) findings when working with a group of third grade students. Englard transitioned these students through solving problems from concrete, in the form of realia, to abstract. The result was that the students who received this problem solving treatment developed their problem solving skills more than a group of students who did not receive the treatment. CGI posits that students should be able to solve problems in the way that makes the most sense to them and that problem solving develops through a concrete to abstract passage (Carpenter et al., 1999). Getting students accustomed to reasoning abstractly and understanding the meaning of numbers and operations in number sentences is important for success with higher level mathematics, especially the Common Core State Standards (Common Core State Standards Initiative, 2010; White & Dauksas, 2012).

Overall, problem solving times decreased as a result of this innovation, though not at a statistically significant rate. The average length it took video recorded dyads to solve daily problem solving questions in the Phase 1 of the innovation compared to Phase 7 decreased by 21.84%. As seen in the video recorded observations, the lowest ability dyad spent less time giggling, looking around, and waiting for the other partner to solve the problem at the end of the innovation than at the beginning of the innovation. The highest ability dyad solved problems more quickly at the end of the innovation by writing faster and doing work individually then comparing answers by looking at each other's papers or saying the answer to each other.

Another way that an increase in efficiency in problem solving was seen in this study was the rate at which solutions were given on the post-assessment. Students immediately stated the answer correctly on 24 out of the 95 total solutions given. They then went on to justify their work by describing what they thought about to solve the problem or stating the number sentence they used to solve the problem. Students only stated the correct answer immediately on five out of the 95 total pre-assessment questions.

Through participation in the guided incremental steps of this study, students became more independent in their problem solving abilities. Nearly all dyads used less words when working with their partner to solve problems at the end of the innovation as compared to the beginning of the innovation. The mean words spoken during the students-at-work portion of the daily problem solving routine decreased 27.76% from Phase 1 to Phase 7, highlighted by the high ability dyad whose words spoken decreased 45.45%. Additionally, students did not rely on their partner to help solve the problem as much, and did not always agree with their partner's answer. When this occurred, students began using their own solution strategy to come up with a different answer. Students were talked out of their solution strategy less often, as seen in the video recorded observations.

When looking at these finding through the constructivist lens, it is easy to see that by participating in the guided incremental steps of this study, students created their own understandings of problem solving and employed the strategies that made sense to them.

First, students did not solely rely on their partner's answer. Toward the end of the innovation, they thought for themselves and disagreements in answers occurred. Next, students made connections between the actions implied by the wording of the problem and the necessary steps to take to solve the problem. For some students, that was the use of realia or manipulatives, for others it was the use of schematic representations, and still for others it was the understanding of the parts of the number sentence. Students used the strategy that made sense to them on the post-assessment, and did it effectively.

Students also learned from others in the classroom, as social development theory and social learning theory describe. The MKO shared during the strategy conference in each lesson and this led to increased complexity of students' solution strategies and understanding of the actions behind solving the problems, as well as increased correctness when solving mathematics word problems. Additionally, working with a partner gave students an opportunity to discuss their problem solving ideas and someone to assist when they were stuck on a problem. Students did not always employ the MKO's or their partner's solution strategy on future problems though. Students took time to understand others' solution strategies, as in the case of the use of number sentences to solve problems, as well as decide if they thought the solution strategy and solution were a better method and answer than what they were employing independently. At the end of the innovation, the post-assessment showed that this innovation process combined with students' innate problem solving abilities and helped them develop their problem solving efficiency and efficacy.

Data Analysis Quality

Data analysis quality was considered when designing and implementing this study, as well as when analyzing the study's data. Of special consideration when analyzing data were reliability, validity, and trustworthiness. Although bias cannot be completely eliminated from a study, deliberate actions were taken to minimize it. First, all students who were available for the study were included. No students were eliminated from the sample due to mathematical ability, English language learning status, socio-economic status, race, nationality, level of parental involvement, age, or gender. Second, how I worked to ensure reliability, validity, and trustworthiness will be described next, as well as noting any limitations to these traits.

Reliability. Reliability can be described as making sure that procedures used in a study are stable across different researchers and within studies (Gibbs, 2007). This study is founded on action research and is meant to impact and influence the teaching in my–the research-practitioner's–classroom. This study is not meant to be generalizable, though it may be replicated in other classrooms, with the understanding that the results found in this study were applicable to only this study (Stringer, 2007).

Care has been taken in the design of both methods and implementation of this research plan so that reliability will be ensured. CGI-style problems were used for the pre-/post-assessment and the daily student problem solving exercises in this study. CGI has been thoroughly researched beginning in the late 1980s (Carpenter et al., 1999).

To insure reliability in the data analysis, multiple safeguards were enlisted. First, intracoder reliability was considered and obtained when analyzing the qualitative data. To do this, pre- and post-assessment qualitative data were coded three times, in three different formats. Codes were then compared to each other and compatibility among the codes was analyzed (Johnson & Christensen, 2004). Next, all transcribed dyads' solution strategy actions and words and 30% of the students' assessment solution strategy words and actions were checked for accuracy by a trained co-analyst. Since the co-analyst was specifically trained by me, intercoder reliability was high, at nearly 90%. Intercoder reliability at 80% or greater is considered reliable (Miles & Huberman, 1994). Next, open codes and axial codes were verified for inclusiveness and accuracy by a trained co-analyst. Quantitative data were also peer-checked. Data entered in Microsoft Word and data analysis tests run in SPSS were verified by a seasoned researcher, checking for accuracy. When all quantitative results were found, an additional researcher reviewed them for accuracy.

Validity. Keeping the results of this study valid took careful planning. Validity is important because without accurate results, this study would serve no one (Gibbs, 2007). Since I served as the researcher and also rated the students' solution strategy and strategy subsets and analyzed all of the data, validity could be a concern. To counter this, carefully designed steps were taken. First, triangulation was used in this study. Having multiple data sources weigh in on findings reduces the chances that inaccurate results will be presented (Creswell, 2009). Also, since students were assigned to my class, and therefore selected for the study, using a stratified random sampling technique and all students present in the classroom during mathematics time participated in the innovation, sampling bias was minimized. This randomization also helped to eliminate confounding variables. Additionally, the short timeframe, the design, and the participants of this study worked in the favor of maintaining a high level of validity. Since the study took place over the course of four months, one would not expect great natural maturation in second grade students' problem solving abilities on these types of CGI-style problems. Also, being a second grade classroom, mortality, or students leaving the study, was not high because students must participate in the day's math activities if they are in the classroom, and only three participants moved during the implementation period. The study was designed so that problem types, names, and numbers used on the pre-/post-assessment were not excessively reused on daily problem solving questions, so testing familiarity was not an issue. Additionally, there was a minimum of four months between the administration of the pre-assessment and post-assessment, which was enough time between test administrations for participants to not recall test questions. No participants commented about having completed these test questions previously while the post-assessment was being administered. Another main validity factor in this study was the video recorded dyads' problem solving lengths and number of words spoken results. Showing that results were not altered, it can be seen that these findings were not statistically significant. Had there been validity issues, the results would be more apt to show favorable results for all aspects of the study.

When analyzing the Video Recorded Observations, the problem of premature typification was addressed in two ways. First, observations were video recorded. This allowed me to review what students said and did while I was transcribing. Second, the formation of ideas and assertions while a researcher views and transcribes observations is a natural process (Erickson, 1986), but one which I tried to minimize as a researcher, even though it is difficult to eliminate them completely (Creswell, 2009). To diminish this effect, I looked for disconfirming cases while reviewing transcripts of the observations. This was important because premature typification would have skewed the assertions I created from the observations (Erickson, 1986). The main drawback of video recorded observations is that the researcher misses out on the contextual situation of the event because the observer cannot see what happened prior to or after the recording or what is going on around the observation site (Erickson, 1986). I was able to mitigate these threats by being in the classroom before, during, and after the video recorded observations, conducting two of the three video recorded observations each week, and focusing the transcription and analysis on what happened within that dyad during only the students-at-work phase of the innovation. What other dyads were doing during this time, and what happened before or after the observation were not included in analysis nor allowed to weigh in on findings. Further, validity was strengthened by me being in the classroom everyday for the innovation. Creswell (2009) describes that the more time the researcher is able to spend in the research setting the deeper, more accurate, and more robust the finding will be.

Trustworthiness. Trustworthiness in qualitative data was a concern in this study because it employed two qualitative data sources, student pre-assessment and post-assessment solution actions and video recorded observations. To ensure trustworthiness, this innovation design, as well as the video recorded observation plan, the assessment plan, the data collection plan, and the data analysis plan were reviewed and critiqued by external auditors (Creswell, 2009). These auditors included a small group of fellow doctoral student researchers and a seasoned mixed-methods researcher.

Degree of Action Plan Implementation

This action plan was fully implemented as organized and planned, with only one alteration; students who were absent from the classroom for a daily problem solving question were not included in that day's percentage correct score. The comparison of the first 20 daily problem solving questions to the last 20 daily problem solving questions was the only data analysis that involved daily scores. This effected 12 out of the 19 participants at least once during the study. The rest of the study ran as designed, with the innovation occurring once daily for the entire 60 days stated in the plan, and students followed the designed implementation steps for each day's lesson as laid out in the study design.

Consequences of Implementation

As a consequence of the implementation, students spoke about reasons why people were doing processes and justified their work more than they did in other subject areas. As evidenced by informal classroom observations, conversation in the classroom developed at a deeper level than had been previously heard in the classroom. Students were more apt to give reasons for their answers, and classmates were more likely to comment on other students' answers in other subjects. Implementing this plan required a reduction in the amount of time spent on the traditional math program, Scott Foresman Mathematics, 2nd grade level. Despite this, students showed a greater interest in math class, seen by student interactions with each other, the number of students participating in the lesson, the number of students completing their math tasks, and an increase in the number of students talking about math class outside of the math period.

CHAPTER 6

CONCLUSION

Results Reflections

Onslow (1991) states that to be an efficient problem solver, students must be able to flexibly use a variety of strategies to solve problems. Students should be able to move from abstract to concrete and have understanding with each method. When students show this, they are mathematically literate. Students in this study used strategies flexibly. They used the strategy they felt most adept with and the strategy they were able to use to come to the correct answer on each problem. Students used their understandings to guide their problem solving strategies, as CGI states (Carpenter et al., 1999). This innovation, with its systematic development from concrete to abstract, allowed students to develop their problem solving understandings and abilities.

Lessons Learned

The favorable results of this study have had an impact on my pedagogy, have raised my expectations for my students, have deepened my beliefs about my students' capabilities, will guide how I will tackle problems in my classroom in the future, and will influence the lessons I design with my coworkers to teach mathematics to our second grade students.

Implications for my future practice. I was very pleased that I tried a new innovation in my classroom, not only because my students exceeded my expectations, but that I pushed myself out of my comfort zone. I am an educator that is willing to try divergent ways of teaching if I feel it will benefit my students, and I normally teach using a constructivist approach, but at times I give up on an idea too early. I may have done that in this case as well if I had not had my research plan and a reason to continue. I remember watching some of my lower ability students complete the daily problems the first two weeks and many of their answers were guesses. Even working with a partner, they were unable to unpack the problems, describe what they were doing, or explain why they did what they did to solve the problem. Though I had thoroughly researched strategies to include in my innovation and was aware of the successes these strategies had in other classrooms, I did not know if my innovation would be successful in my classroom. It was not until the 13^{th} day of implementation that my two lowest ability dyads were able to successfully solve the day's problem and explain what they were doing. At that point I knew I was on the right path. There have been years in the past when even at the end of the school year, my students with low mathematics ability could not successfully solve like-worded problems. In hindsight, I am thankful that it was necessary for me to continue with my innovation.

Taking what I have learned with me in the future, I plan to continue teaching through social learning, social development, and constructivist frameworks. Giving students the ability to work together and to learn from each other will continue to guide lessons I prepare for my students (Bandura, 1977). Allowing students to share their thinking and listen to others' thinking will continue to be integral parts of students' days (Vygotsky, 1978). Teaching with an expectation that students truly listen to classmates' thoughts and ideas, relate others' ideas to their own, and work cooperatively to learn together will help my students continue to succeed in mathematics problem solving and in new areas (Vygotsky, 1962).

Implications for my future research. Completing this dissertation study has given me a basis for future studies, and I have already started a new action research project which involves the entire second grade team. When conducting the video recorded weekly observations, other dyads in the classroom showed strong interest in being video recorded. Because I observed that no students gave up on the problem they were solving while they were being video recorded, I thought that using video cameras in the classroom might be an effective strategy for motivating students to persevere when solving challenging math problems. I applied for and received a grant that allowed me to purchase seven Flip cameras for my grade level. I have structured my new study in a similar fashion to the study I just completed, using a pre- and post-test design, but this time, I am focusing on researching the interplay between students using video cameras to record their work, perseverance, and problem solving abilities. Again, CGI-style problem types are being used, but numbers with greater values are included in the problems. This research project began in January and will conclude in May and has 80 participants.

Implications for participants. By participating in this study, all students showed growth in their ability to correctly solve CGI-style mathematics word problems. Further, many students became more reliant on mental strategies and were able to use visualization to solve problems with understanding. Additionally, they learned to dialogue and discuss with their classmates, in both paired and whole group settings. Just as Hartweg and Heisler (2007) found in their study, students showed respect when discussing others' problem solving, including when errors were found. They worked as a group to develop mathematical understandings from the misconception. This is an important development, especially for students from low socio-economic homes; a development in which the students, as well as future teachers and future employers, will benefit (Kilpatrick & Swafford, 2002; Lester Jr. & Charles, 2003; NCTM, 2000, 2004; Sutton & Krueger, 2002).

But one of the greatest benefits I have seen come from this study was something that was not directly measured. I have observed that student motivation has improved as a result of the study. During this study, students were excited for the daily problem solving time, and even now still cheer when they see a word problem. With some extra enthusiasm on my part after the study ended, this has carried over to other subject areas as well. I observe on a daily basis, students who are engaged, ask questions of each other, give their thoughts about what we are studying, and have a positive attitude toward learning new things. Students informally start class discussions more in the classroom, and students feel at ease when sharing ideas with the class. This is important to me as a teacher because Mohd, Mahmood, and Ismail (2011) describe that there is a positive correlation between students' attitudes about problem solving and their overall mathematics achievement. Additionally, cognitive development occurs faster in students who are motivated and engaged in their learning (Kamii & Rummelsburg, 2008), and higher levels of participation and attention relate to higher standardized test scores (Alexander et al., 1993; Duncan et al., 2007; Finn et al., 1995; Horn & Packard, 1985; McClelland et al., 2000; Schaefer & McDermott, 1999; Tramontana et al., 1988; Yen et al., 2004). Continuing to promote these ways of being as students will be an important aspect of success in my classroom.

New Questions

It was clear in this study that student participants grew in many different ways. Most notably, and what I set out to help them improve, their problem solving abilities improved. Their understanding of how to solve problems increased, their skills to check the reasonableness of their answers increased, their ability to try different strategies increased, their ability to internalize mathematics problems increased, their ability to use higher order problem solving hierarchy methods increased, and their independent thinking increased. I understand that there were parts of this innovation that were not directly studied that very likely played a large role in contributing to these improvements. First, every day students interacted with an MKO dyad during the strategy conference. This MKO dyad shared their solution strategies with the class and then allowed classmates to ask questions and have discussions about their solution strategies. Through my informal observations, I found that the discussions the class had developed throughout the implementation period. At first, students listening to the MKO appeared to sit patiently while the MKOs described how they solved the problem. The discussion that followed basically revolved around students asking if they could go to the front of the class and show how they solved the problem, even if their solution strategy was exactly the same as what was just shown. Throughout the course of the innovation, students began listening and trying to understand what their classmates were saying. The discussion became robust, with questions and comments about the solution strategies shared. Students were able to compare their solution strategy with the MKO dyad's solution strategy and were able to identify if they had an original idea that the class would benefit from hearing about or if their idea was very similar to the MKOs'. At the end of the innovation period, students were making agree/disagree statements with justifications, asking what would happen to the answer if the numbers were different, suggesting alternative equations that could be used to solve the problem, and suggesting base ten strategies that could be used to mentally solve problems. I believe that future research in how students in the class viewed the role of the MKO and the affect of the MKO on their problem solving skills, including strategies used and correctness of answers, would be beneficial. My research has shown that my innovation as a whole was effective in increasing students' problem solving abilities, but it does not identify the direct parts of the innovation that benefitted students the most or describe students' perceptions of the strategy conference portion of the innovation. If the strategy conference is shown to be of major importance to developing students' abilities, then it will likely also be beneficial to my students in other areas of mathematics instruction. This would directly coincide with the goals of the developers of the Common Core State Standards (White & Dauksas, 2012) and what NCTM deems as effective mathematics pedagogical strategies (NCTM, 2000).

Conclusion

Through this study I learned quite a bit about myself as an educator and researcher and I learned more about how my children learn mathematics. More importantly than these things, I feel that I learned what can be possible for my Title I students. It was amazing for me to see what a systematic, research based practice conducted over an extended period can guide my students into being able to do. In the end, the students made an outstanding effort to move outside of their comfort zones and participate in authentic class discussions. They thought about what classmates were saying, which is not always easy (Reinhart, 2000), especially for a non-native English speaking 7 year old. When discussing what I have witnessed at San Marcos Elementary at the beginning of this paper, I asked two deep questions about the mathematics program at the school–Why were these stagnant test scores continuing to occur? What happens if students lack the background experiences and knowledge needed for mental imagery to create schematic representations of problem situations? Through this study I did my best to help develop the problem solving skills of my second grade students so that when they move on to the intermediate elementary grades these questions will still not be lingering for them. Action research is designed to positively impact the people that the practitioner works with (Stringer, 2007), and in this case I feel satisfied that I did. I hope my students do as well.

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APPENDIX A

PRE-ASSESSMENT/POST-ASSESSMENT QUESTIONS

Problem Solving Pre-/Post-Assessment

Join, Change Unknown Problem

1. Robin had 4 toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?

Join, Start Unknown Problem

2. Deborah had some books. She went to the library and got 3 more books. Now she has 8 books altogether. How many books did she start with?

Separate, Change Unknown Problem

3. Roger had 13 stickers. He gave some to Colleen. He has 4 stickers left. How many stickers did he give to Colleen?

Separate, Start Unknown Problem

4. Some birds were sitting on a wire. 3 birds flew away. There were 8 birds still sitting on the wire. How many birds were sitting on the wire before the 3 birds flew away?

Compare, Referent Unknown Problem

5. Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 12, 16, 17, 19, & 29), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann.

APPENDIX B

SOLUTION STRATEGY RECORDING FORM

Solution Strategy Recording Form

Student ID Number ____

Date of Assessment ____

Pre-assessment Post-assessment

Problem 1: Robin had 4 toy cars. How many more toy cars does she need to get for her birthday to have 11 toy cars all together?

Student actions: ____

____

____

Student's answer: ____

Is the student's answer correct? yes no

Solution strategies: Circle the strategies the student used.

Solution Strategy	Direct Modeling	Counting	Number Facts
Solution Strategy Subset	Joining All	Counting On From First	Derived Fact
	Joining To	Counting On From Larger	Recalled Fact
	Separating From	Counting On To	
	Separating To	Counting Down	
	Matching	Counting Down To	
	Trial and Error		

Problem 2: Deborah had some books. She went to the library and got 3 more books. Now she has 8 books altogether. How many books did she start with?

Student actions: ____

____

____

____

____

____

____

____

Student's answer: ____

Is the student's answer correct? yes no

Solution strategies: Circle the strategies the student used.

Solution Strategy	Direct Modeling	Counting	Number Facts
Solution Strategy Subset	Joining All	Counting On From First	Derived Fact
	Joining To	Counting On From Larger	Recalled Fact
	Separating From	Counting On To	
	Separating To	Counting Down	
	Matching	Counting Down To	
	Trial and Error		

Problem 3: Roger had 13 stickers. He gave some to Colleen. He has 4 stickers left. How many stickers did he give to Colleen?

Student actions: ____

____

____

____

____

____

____

____

Student's answer: ____

Is the student's answer correct? yes no

Solution strategies: Circle the strategies the student used.

Solution Strategy	Direct Modeling	Counting	Number Facts
Solution Strategy Subset	Joining All	Counting On From First	Derived Fact
	Joining To	Counting On From Larger	Recalled Fact
	Separating From	Counting On To	
	Separating To	Counting Down	
	Matching	Counting Down To	
	Trial and Error		

Problem 4: Some birds were sitting on a wire. 3 birds flew away. There were 8 birds still sitting on the wire. How many birds were sitting on the wire before the 3 birds flew away?

Student actions: ____

____

____

____

____

____

____

____

Student's answer: ____

Is the student's answer correct? yes no

Solution strategies: Circle the strategies the student used.

Solution Strategy	Direct Modeling	Counting	Number Facts
Solution Strategy Subset	Joining All	Counting On From First	Derived Fact
	Joining To	Counting On From Larger	Recalled Fact
	Separating From	Counting On To	
	Separating To	Counting Down	
	Matching	Counting Down To	
	Trial and Error		

Problem 5: Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Student actions: ____

____

____

____

____

____

____

Student's answer: ____

Is the student's answer correct? yes no

Solution strategies: Circle the strategies the student used.

Solution Strategy	Direct Modeling	Counting	Number Facts
Solution Strategy Subset	Joining All	Counting On From First	Derived Fact
	Joining To	Counting On From Larger	Recalled Fact
	Separating From	Counting On To	
	Separating To	Counting Down	
	Matching	Counting Down To	
	Trial and Error		

Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 12, 16, 17, 19, & 29), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann.

APPENDIX C

PHASE 1, 2, AND 3 PROBLEM SOLVING QUESTIONS

Phase 1, Day 1: (Part-Part-Whole, Whole Unknown)

Francine has 3 red markers and 5 blue markers. How many markers does she have?

Phase 1, Day 2: (Separate, Results Unknown)

There were 8 seals playing. 3 seals swam away. How many seals were still playing?

Phase 1, Day 3: (Compare, Difference Unknown)

Megan has 3 stickers. Randy has 8 stickers. How many more stickers does Randy have than Megan?

Phase 1, Day 4: (Join, Results Unknown)

Maggie had 7 pencils. She bought 4 more from the school store. How many pencils does she have now?

Phase 1, Day 5: (Separate, Change Unknown)

Daisy had 13 marbles. She gave some to Luke. Now she has 5 marbles left. How many marbles did Daisy give to Luke?

Phase 1, Day 6: (Compare, Referent Unknown)

Lilly found 8 shells. She has 2 more shells than James. How many shells does James have?

Phase 1, Day 7: (Join, Change Unknown)

Felicity has 3 raisins. How many more raisins does she need to have 10 raisins altogether?

Phase 1, Day 8: (Part-Part-Whole, Part Unknown)

Humberto loves to read books. He has 9 books in all. 5 of his books are picture books and the rest are chapter books. How many chapter books does Humberto have?

Phase 1, Day 9: (Join, Start Unknown)

Val had some erasers. Herb gave her 6 more. Val now has 9 erasers. How many erasers did Val have to start with?

Phase 1, Day 10: (Join, Results Unknown)

Shanique made a book. She used 10 pieces of her own paper. She needed more paper so she got 4 more pieces from her mom. How many pieces of paper did Shanique use in her book?

Phase 2, Day 1: (Join, Start Unknown)

Olivia saw some ladybugs on a leaf. 3 more flew up and landed on the leaf. Now there are 7 ladybugs on the leaf. How many ladybugs were on the leaf to start?

Phase 2, Day 2: (Part-Part-Whole, Whole Unknown)

Maxwell Jones collects shapes. He has 6 triangles and 5 rectangles. How many shapes does he have in all?

Phase 2, Day 3: (Compare, Difference Unknown)

Vinnie has 7 blue flowers and 9 red flowers. How many more red flowers does Vinnie have than blue flowers?

Phase 2, Day 4: (Part-Part-Whole, Part Unknown)

Galaxy Comic Books Store sells only expensive Spiderman and Batman comic books. They have a total of 10 comic books in their store. 6 of the comic books are Batman and the rest are Spiderman. How many Spiderman comic books does Galaxy Comic Books Store have?

Phase 2, Day 5: (Compare, Referent Unknown)

Mandy has 12 potato chips. She has 4 more chips than her brother, Josue. How many potato chips does Josue have?

Phase 3, Day 1: (Compare, Difference Unknown)

Hildy has 12 cupcakes. Martin has 13 cupcakes. How many more cupcakes does Martin have than Hildy?

Phase 3, Day 2: (Separate, Change Unknown)

Rebecca had 14 pea plants. She overwatered them and some died. She now has 9 living pea plants. How many pea plants died?

Phase 3, Day 3: (Join, Start Unknown)

Petunia had some paperclips. She found 3 more on the floor. Now she has 11 paperclips. How many paperclips did Petunia originally have?

Phase 3, Day 4: (Part-Part-Whole, Part Unknown)

Lyle has 15 video games. 5 are hunting games and the rest are driving games. How many driving video games does Lyle have?

Phase 3, Day 5: (Separate, Results Unknown)

The Willis family had 5 cars. One got in an accident and the family had to get rid of it. How many cars does the Willis family have left?

Phase 3, Day 6: (Compare, Quantity Unknown)

Patsy has 16 rings. Lainey has 5 more rings than Patsy. How many rings does Lainey have?

Phase 3, Day 7: (Separate, Start Unknown)

Gracie had some pictures of her friends in her purse. She lost 3 of the pictures. Gracie now has 6 pictures left. How many pictures of her friends did Gracie have to start?

Phase 3, Day 8: (Compare, Referent Unknown)

Steven has 12 bite-size cookies. He has 8 more cookies than Chantel. How many bite-size cookies does Chantel have?

Phase 3, Day 9: (Part-Part-Whole, Whole Unknown)

Jasmine has 12 pieces of watermelon bubblegum and 14 pieces of strawberry bubblegum. How many pieces of gum does she have?

Phase 3, Day 10: (Compare, Quantity Unknown)

Hareem has 1 ant in his ant farm. Trudy has 13 more ants than Hareem. How many ants does Trudy have on her ant farm?

Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 7-29), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann.

APPENDIX D

MATHEMATICS PROBLEM SOLVING JOURNAL

My Mathematics Problem Solving Journal

By ______________________________

Phase 4, Day 1: Please solve this problem using manipulatives and then a schematic representation. Write your answer in the blank.

Stacy had 15 erasers. She gave 3 to Jeremy. How many erasers does Stacy have left?

Answer: Stacy has ___________ erasers left.

Phase 4, Day 2: Please solve this problem using manipulatives and then a schematic representation. Write your answer in the blank.

Clara saw 13 butterflies in her garden. Some flew away. Now she sees 6 butterflies left. How many butterflies flew away?

Answer: _________ butterflies flew away.

Phase 4, Day 3: Please solve this problem using manipulatives and then a schematic representation. Write your answer in the blank.

Joyce has 11 seashells. Juan has 9 seashells. How many more seashells does Joyce have than Juan?

Answer: Joyce has __________ more seashells than Juan.

Phase 4, Day 4: Please solve this problem using manipulatives and then a schematic representation. Write your answer in the blank.

Flora found 6 beautiful fall leaves. Then she found 4 more leaves. How many leaves does Flora have altogether?

Answer: Flora has __________ leaves.

Phase 4, Day 5: Please solve this problem using manipulatives and then a schematic representation. Write your answer in the blank.

Dale has 5 quarters. How many more quarters does he need to have 12 quarters altogether?

Answer: Dale needs __________ more quarters.

Phase 5, Day 1: Please solve this problem using a schematic representation and write your answer in the blank.

Julio had 9 envelopes to take to the post office. His mom gave him 5 more envelopes. How many envelopes did he have then?

Answer: Julio had _______________ envelopes to take to the post office.

Phase 5, Day 2: Please solve this problem using a schematic representation and write your answer in the blank.

Colleen had 12 guppies. She gave 5 guppies to Roger.

How many guppies does Colleen have left?

Answer: Colleen has __________ guppies left.

Phase 5, Day 3: Please solve this problem using a schematic representation and write your answer in the blank.

Mark has 6 toy mice. Joy has 11 mice. Joy has how many more toy mice than Mark?

Answer: Joy has ________ more toy mice than Mark.

Phase 5, Day 4: Please solve this problem using a schematic representation and write your answer in the blank.

Bryce had 3 pieces of candy. Rosa gave him some more candy. Now Bryce has 9 pieces of candy. How many pieces of candy did Rosa give him?

Answer: Rosa gave Bryce _________ pieces of candy.

Phase 5, Day 5: Please solve this problem using a schematic representation and write your answer in the blank.

Lisa had some comic books. She went to the store and bought 5 more comic books. Now she has 11 comic books altogether. How many comic books did she have to start with?

Answer: Lisa had _________ comic books to start.

Phase 5, Day 6: Please solve this problem using a schematic representation and write your answer in the blank.

There were 4 clean cups in the cupboard. Jimmy's family used some of the cups. Now there is 1 cup in the cupboard. How many cups did Jimmy's family use?

Answer: Jimmy's family used _________ cups.

Phase 5, Day 7: Please solve this problem using a schematic representation and write your answer in the blank.

Linda has 5 markers. How many more markers does she need to have 13 markers altogether?

Answer: Linda needs _________ more markers.

Phase 5, Day 8: Please solve this problem using a schematic representation and write your answer in the blank.

Rolando has 14 stuffed animals. Horace has 6 stuffed animals. How many more stuffed animals does Rolando have than Horace?

Answer: Rolando has _________ more stuffed animals than Horace.

Phase 5, Day 9: Please solve this problem using a schematic representation and write your answer in the blank.

Gibby had 16 video games. He has 9 more video games than Veronica. How many video games does Veronica have?

Answer: Veronica has _________ video games.

Phase 5, Day 10: Please solve this problem using a schematic representation and write your answer in the blank.

Blaze has 12 gray socks and 7 red socks. How many socks does he have?

Number sentence: ______________________________

Answer: Blaze has __________ socks.

Phase 6, Day 1: Please solve this problem using a schematic representation and a number sentence. Write your answer in the blank.

CiCi had 14 books. She returned 4 to the library. How many books does CiCi have left?

Number sentence: ______________________________

Answer: CiCi has _________ books left.

Phase 6, Day 2: Please solve this problem using a schematic representation and a number sentence. Write your answer in the blank.

Jasmine had some grapes. She gave 3 to Marco. Now she has 8 grapes left. How many grapes did Jasmine have to start with?

Number sentence: ______________________________

Answer: Jasmine started with _________ grapes.

Phase 6, Day 3: Please solve this problem using a schematic representation and a number sentence. Write your answer in the blank.

Caleb has 5 miniature candy bars. Sue has 8 more than Caleb. How many miniature candy bars does Sue have?

Number sentence: ______________________________

Answer: Sue has __________ miniature candy bars.

Phase 6, Day 4: Please solve this problem using a schematic representation and a number sentence. Write your answer in the blank.

Karen has 12 marbles. 4 are blue and the rest are orange. How many orange marbles does Karen have?

Number sentence: ______________________________

Answer: Karen has _________ orange marbles.

Phase 6, Day 5: Please solve this problem using a schematic representation and a number sentence. Write your answer in the blank.

Frank had some pet lizards. Annie gave him 7 more lizards. Now he has 13 lizards. How many pet lizards did Frank have before Annie gave him more?

Number sentence: ______________________________

Answer: Frank had __________ pet lizards.

Phase 7, Day 1: Please solve this problem using a number sentence. Write your answer in the blank.

Jane has 7 purple paintbrushes and 11 yellow paintbrushes. How many paintbrushes does she have?

Number sentence: ______________________________

Answer: Jane has __________ paintbrushes.

Phase 7, Day 2: Please solve this problem using a number sentence. Write your answer in the blank.

Flo has a chicken coop. She first gathered 9 eggs. Then her hens laid 3 more eggs. How many eggs did she have then?

Number sentence: ______________________________

Answer: Flo has __________ eggs.

Phase 7, Day 3: Please solve this problem using a number sentence. Write your answer in the blank.

Wilma has 7 jelly bracelets. How many more jelly bracelets does she need to get from her family for Christmas to have 10 jelly bracelets in all?

Number sentence: ______________________________

Answer: Wilma needs to get __________ more jelly bracelets.

Phase 7, Day 4: Please solve this problem using a number sentence. Write your answer in the blank.

Paul had 20 fireflies in a jar. He let 9 go. How many fireflies does Paul have left?

Number sentence: ______________________________

Answer: Paul has __________ fireflies left.

Phase 7, Day 5: Please solve this problem using a number sentence. Write your answer in the blank.

Destiny has 17 smelly stickers. 13 are strawberry scented and the rest are chocolate scented. How many chocolate scented smelly stickers does Destiny have?

Number sentence: ______________________________

Answer: Destiny has __________ chocolate scented stickers.

Phase 7, Day 6: Please solve this problem using a number sentence. Write your answer in the blank.

Sammy loves to collect colorful buttons. He had 12 buttons. He gave some to Wendy. He has 2 buttons left. How many buttons did he give to Wendy?

Number sentence: ____________________________

Answer: Sammy gave __________ buttons to Wendy.

Phase 7, Day 7: Please solve this problem using a number sentence. Write your answer in the blank.

Leslie has 8 carrot sticks on her lunch tray. Kevin has 12 carrot sticks. Kevin has how many more carrot sticks than Leslie?

Number sentence: ____________________________

Answer: Kevin has __________ more carrot sticks than Leslie.

Phase 7, Day 8: Please solve this problem using a number sentence. Write your answer in the blank.

LaDova had some pretzels. Her teacher gave her 9 more pretzels at snack time. Then she had 13 pretzels. How many pretzels did LaDova have before snack time?

Number sentence: ____________________________

Answer: LaDova had ___________ pretzels before snack time.

Phase 7, Day 9: Please solve this problem using a number sentence. Write your answer in the blank.

Sam had 24 flowers. He picked 3 more. How many flowers did he have then?

Number sentence: ______________________________

Answer: Sam now has __________ flowers.

Phase 7, Day 10: Please solve this problem using a number sentence. Write your answer in the blank.

Bill had some baseball cards. He gave 2 to Spencer. Now Bill has 21 cards left. How many baseball cards did Bill have to start with?

Number sentence: ____________________________

Answer: Bill started with __________ baseball cards.

Phase 7, Day 11: Please solve this problem using a number sentence. Write your answer in the blank.

Ellen had 3 tomatoes. She picked 5 more tomatoes. How many tomatoes does Ellen have now?

Number sentence: ______________________________

Answer: Ellen now has __________ tomatoes.

Phase 7, Day 12: Please solve this problem using a number sentence. Write your answer in the blank.

Deshawn has 13 pencils. He has 5 more pencils than Tricia. How many pencils does Tricia have?

Number sentence: ________________________________

Answer: Tricia has ____________ pencils.

Phase 7, Day 13: Please solve this problem using a number sentence. Write your answer in the blank.

Chuck has 3 peanuts. Clara gave him some more peanuts. Now Chuck has 8 peanuts. How many peanuts did Clara give him?

Number sentence: ____________________________

Answer: Clara gave Chuck _________ peanuts.

Phase 7, Day 14: Please solve this problem using a number sentence. Write your answer in the blank.

Ray has 15 fish. 9 are goldfish and the rest are angelfish. How many angelfish does Ray have?

Number sentence: ______________________________

Answer: Ray has _________ angelfish.

Phase 7, Day 15: Please solve this problem using a number sentence. Write your answer in the blank.

Paco has 8 bouncy balls. Nina has 3 more than Paco. How many bouncy balls does Nina have?

Number sentence: ______________________________

Answer: Nina has ___________ bouncy balls.

Adapted from *Children's Mathematics: Cognitively Guided Instruction* (p. 7-29), T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, & S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright 1999 by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, Susan B. Empson.

APPENDIX E

DAILY ANSWER RECORDING SLIP SAMPLE

Name ______________________ Date ______________

Francine has ____________ markers.

- -

Name ______________________ Date ______________

Francine has ____________ markers.

- -

Name ______________________ Date ______________

Francine has ____________ markers.

- -

Name ______________________ Date ______________

Francine has ____________ markers.

- -

Name ______________________ Date ______________

Francine has ____________ markers.

- -

Name ______________________ Date ______________

Francine has ____________ markers.

Note. This form will be cut into strips and each student will complete one slip. This form is for the first day of the innovation.

APPENDIX F

STUDENT ANSWER CORRECTNESS CHART

Pre-assessment / Post-assessment							
Student's ID #	Question Number					Number Correct	% Correct
	1	2	3	4	5		

APPENDIX G

VIDEO RECORDING OBSERVATION PROTOCOL

Date:

Time:

Dyad number:

Problem:

Descriptive Notes (dialogue, events, strategies, movements, etc.)	Reflective Notes (thoughts, speculations, biases, feelings, impressions, etc.)

Adapted from *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (p. 180-181), by J. W. Creswell, 2009, New Delhi, India: Sage. Copyright 2009 by Sage Publications.

APPENDIX H

STUDENT ANSWER SOLUTION STRATEGY CHART

Pre-assessment and Post-assessment Comparison										
	Question Number									
	1		2		3		4		5	
Student’s ID #	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post

APPENDIX I

DAILY PROBLEM SOLVING ANSWER CHART

Problem Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Correct Answer													
Student’s ID #													
% Correct													

APPENDIX J

VIDEO RECORDED OBSERVATION DYADS TRANSCRIPTION DATA CHART

Low								
Date	8-1	8-8	8-15	8-22	8-29	9-5	9-12	9-19
Correct Y/N								
Problem Type								
# of Words Said								
# of Words Said Without Problem								
Length								
Phase/Day	P1/D3	P1/D8	P2/D3	P3/D3	P3/D8	P4/D2	P5/D2	P5/D7
Average Length per Phase								
Medium								
Date	8-1	8-8	8-15	8-22	8-29	9-5	9-12	9-19
Correct Y/N								
Problem Type								
# of Words Said								
# of Words Said Without Problem								
Length								
Phase/Day	P1/D3	P1/D8	P2/D3	P3/D3	P3/D8	P4/D2	P5/D2	P5/D7
Average Length per Phase								

High								
Date	8-1	8-8	8-15	8-22	8-29	9-5	9-12	9-19
Correct Y/N								
Problem Type								
# of Words Said								
# of Words Said Without Problem								
Length								
Phase/Day	P1/D3	P1/D8	P2/D3	P3/D3	P3/D8	P4/D2	P5/D2	P5/D7
Average Length per Phase								

APPENDIX K

PRE- & POST-ASSESSMENT SOLUTION TRANSCRIPTION CHART

Student ID #:

Pre-assessment or Post-assessment:

Question 1: Did the student answer the problem correctly? ___________
Field Notes:

Question 2: Did the student answer the problem correctly? ___________
Field Notes:

Question 3: Did the student answer the problem correctly? ___________
Field Notes:

Question 4: Did the student answer the problem correctly? ___________
Field Notes:

Question 5: Did the student answer the problem correctly? ___________
Field Notes:

APPENDIX L

CATEGORIES PRE- AND POST-ASSESSMENT SOLUTION STRATEGIES

Code	Category	Definition	Examples

APPENDIX M

CATEGORIZED VIDEO RECORDED OBSERVATION DATA FORM

Code	Category	Definition	Examples

APPENDIX N

COMPLETED STUDENT ANSWER SOLUTION STRATEGY CHART

Student's ID #	Question Number									
	1		2		3		4		5	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	1	2	1	3	2	2	1	3	1	2
2	1	2	1	2	1	1	1	2	1	1
3	0	1	0	3	0	1	0	1	0	3
4	1	1	1	3	3	3	1	3	1	1
5	1	3	1	3	1	1	1	1	1	1
6	1	2	3	3	2	1	3	3	1	3
7	1	3	1	1	1	1	1	1	1	1
8	1	1	1	2	1	1	1	1	1	1
9	2	1	1	3	1	1	1	3	1	1
10	1	1	3	3	3	1	2	1	1	1
11	1	1	0	1	0	2	0	3	1	1
12	1	1	1	1	1	3	1	3	3	2
13	1	3	3	3	3	3	3	3	3	3
14	1	3	1	3	1	3	1	3	1	1
15	1	3	1	3	1	3	1	1	1	1
16	1	2	1	2	1	2	1	2	1	1
17	1	3	1	3	1	3	1	3	1	3
18	1	3	1	3	1	1	1	3	1	2
19	2	3	3	3	1	3	2	3	1	3
Most Common Strategy Used	1	3	1	3	1	1	1	3	1	1

Note. 0 = Guess or no strategy used; 1 = Direct Modeling strategy; 2 = Counting strategy; 3 = Number Facts strategy.

APPENDIX O

COMPLETED STUDENT ANSWER SOLUTION STRATEGY AND STRATEGY SUBSET CHART

Student's ID #	Question Number									
	1		2		3		4		5	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	1-3	2-3	1-5	3-2	2-1	2-3	1-6	3-1	1-5	2-3
2	1-2	2-3	1-4	2-3	1-4	1-4	1-3	2-2	1-5	1-6
3	0	1-2	0	3-2	0	1-4	0	1-6	0	3-2
4	1-2	1-2	1-1	3-2	3-1	3-2	1-3	3-2	1-5	1-3
5	1-2	3-2	1-1	3-2	1-3	1-4	1-3	1-6	1-6	1-3
6	1-2	2-3	3-2	3-1	2-5	1-4	3-2	3-1	1-1	3-2
7	1-3	3-1	1-4	1-2	1-3	1-3	1-1	1-6	1-6	1-3
8	1-6	1-3	1-1	2-1	1-4	1-3	1-3	1-3	1-1	1-3
9	2-3	1-2	1-6	3-2	1-3	1-4	1-3	3-1	1-6	1-5
10	1-3	1-2	3-2	3-2	3-2	1-4	2-4	1-6	1-6	1-5
11	1-1	1-3	0	1-2	0	2-5	0	3-2	1-1	1-5
12	1-2	1-2	1-6	1-2	1-3	3-2	1-3	3-2	3-2	2-5
13	1-2	3-1	3-2	3-2	3-1	3-2	3-1	3-1	3-2	3-2
14	1-2	3-1	1-6	3-2	1-3	3-2	1-3	3-2	1-3	1-3
15	1-2	3-1	1-2	3-2	1-4	3-2	1-1	1-6	1-6	1-5
16	1-2	2-3	1-2	2-2	1-4	2-4	1-6	2-3	1-5	1-5
17	1-2	3-2	1-2	3-2	1-3	3-2	1-3	3-2	1-2	3-2
18	1-1	3-2	1-1	3-2	1-1	1-4	1-3	3-2	1-1	2-4
19	2-3	3-2	3-2	3-2	1-3	3-2	2-1	3-1	1-6	3-2
Most Common Strategy Used	Direct Modeling, Joining To	Direct Modeling, Joining To	Direct Modeling, Joining All and Number Facts, Recalled Facts	Number Facts, Recalled Fact	Direct Modeling, Separating From	Direct Modeling, Separating To and Number Fact, Recalled Fact	Direct Modeling, Separating From	Number Facts, Recalled Fact	Direct Modeling, Trial and Error	Direct Modeling, Separating From and Direct Modeling, Matching and Number Facts, Recalled Fact

APPENDIX P

ARIZONA STATE UNIVERSITY INSTITUTIONAL REVIEW BOARD APPROVAL FORM

ASU Knowledge Enterprise Development

Office of Research Integrity and Assurance

o:	Ronald Zambo FAB
From:	Mark Roosa, Chair Soc Beh IRB
Date:	05/29/2012
Committee Action:	**Exemption Granted**
IRB Action Date:	05/29/2012
IRB Protocol #:	1205007785
Study Title:	Mathematics in a Second Grade Classroom

The above-referenced protocol is considered exempt after review by the Institutional Review Board pursuant to Federal regulations, 45 CFR Part 46.101(b)(1) .

This part of the federal regulations requires that the information be recorded by investigators in such a manner that subjects cannot be identified, directly or through identifiers linked to the subjects. It is necessary that the information obtained not be such that if disclosed outside the research, it could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects' financial standing, employability, or reputation.

You should retain a copy of this letter for your records.

APPENDIX Q

CHANDLER UNIFIED SCHOOL DISTRICT INSTITUTIONAL REVIEW BOARD

APPROVAL FORM

CUSD Institutional Review Board

Date: June 29, 2012

To: Amy Spilde

Cc: IRB file

From: Research Committee

Re: Acceptance of research project/proposal

Dear Amy Spilde,

This letter is notification that your research proposal to conduct action-based research on the teaching of mathematics problem solving strategies within the Chandler Unified School District has been approved. You may conduct your research as outlined in your study with the stipulation that any changes to your protocol must be submitted to the CUSD IRB and receive approval before they are used with students.

Please note that the Principal Investigator is responsible for 1) complying with human subjects research regulations, 2) retaining signed consents by all subjects unless a waiver is granted, 3) notifying the IRB of any and all modifications (amendments) to the protocol and consent form and submitting them to the IRB for approval before implementation and 4) supplying a final report to the district.

Sincerely,

Nicolle Karantinos

Nicolle Karantinos, Ed.D.
Director of Curriculum
IRB Representative